UCL CENTRE FOR ADVANCED SPATIAL ANALYSIS

# WORKING PAPERS SERIES

## Paper 133 - April 08

## Constructing and Implementing an Agent-Based Model of Residential Segregation through Vector GIS

ISSN 1467-1298

casa

Centre for Advanced Spatial Analysis University College London 1 - 19 Torrington Place Gower St London WC1E 7HB
Tel: +44 (0)20 7679 1782 casa@ucl.ac.uk www.casa.ucl.ac.uk

# Constructing and Implementing an Agent-Based Model of Residential Segregation through Vector GIS

Andrew T Crooks[1]

Email: andrew.crooks@ucl.ac.uk

## Abstract

In this paper, we present a geographically explicit agent-based model, loosely coupled with vector GIS, which explicitly captures and uses geometrical data and socio economic attributes in the simulation process. The ability to represent the urban environment as a series of points, line and polygons not only allows one to represent a range of different sized features such as houses or larger areas portrayed as the urban environment but is a move away from many agent-based models utilising GIS which are rooted in grid-based structures. We apply this model to the study of residential segregation, specifically creating a Schelling (1971, 1978) type of model within a hypothetical cityscape, thus demonstrating how this approach can be used for linking vector-based GIS and agent-based modelling. A selection of simulation experiments are presented, highlighting the inner workings of the model and how aggregate patterns of segregation can emerge from the mild tastes and preferences of individual agents interacting locally over time. Furthermore, the paper suggests how this model could be extended and demonstrates the importance of explicit geographical space in the modelling process.

**Keywords:** *Agent-Based Modelling, GIS, Residential Segregation, Repast*

## 1: Introduction

Agent-based modelling (ABM) enables us to simulate the individual actions of diverse agents and measure the resulting system behaviour and outcomes over time. This means that agent-based models can be useful tools for studying the effects of processes that operate at multiple scales and organisational levels (Brown, 2006). Furthermore, as with computer modelling in general, such models allow us to test different ideas and theories of urban change in the safe environment of the computer, therefore allowing scientists to understand urban phenomena through analysis and

[1] Centre for Advanced Spatial Analysis (CASA), University College London, 1-19 Torrington Place, London, WC1E 7HB. 15th April 2008.

experimentation, a traditional goal of science. Although agent-based models have been developed for a diverse range of applications (see Castle and Crooks, 2006 for a recent review), the use of ABM for experimenting and exploring geographical phenomena, specifically linking it to geographical information systems (GIS) is still in its infancy (see Gimblett, 2002; Benenson and Torrens, 2004; Parker, 2005 for some recent applications). The advantage of linking the two allows agent-based modellers to simulate agents related to actual geographic locations, thus allowing us to think about how objects or agents and their aggregations interact and change in space and time (Batty, 2005a). It also provides the ability to model the emergence of phenomena through the individual interaction of features within a GIS over space and time (Najlis and North, 2004).

Many of the applications linking GIS and ABM focus on representing space as a series of discrete cells (e.g. Gimblett *et al.*, 2002), and while these agent-based models have provided valuable insights into urban phenomena especially as they can capture geographic detail, they miss geometric detail. This area is critical to good applications but is barely touched upon in the literature (Batty, 2005b) with a few exceptions (e.g. Benenson *et al.*, 2002). The ability to represent the world as a series of points, lines and polygons allows the inclusion of geometry into the modelling process, thereby allowing for different sizes of features such as houses and roads, for example, to be portrayed and how these features might effect the simulation outcomes depending on the processes being modelled. Additionally the inclusion of geometry allows us to make agent-based models more realistic than as series of discrete regular cells when representing the urban environment.

This paper presents an agent-based model loosely coupled with a vector GIS which explicitly captures and uses geometrical data and related attributes in the simulation process. To highlight this, the model is applied to the study of segregation. The remainder of this paper is as follows: first there is a brief description of segregation and a discussion of agent-based models that have been created to study and explore this phenomenon. Second, the basic model is then introduced focusing on the underlying mechanisms and the use of vector-based GIS data. Third, results from various experiments are presented highlighting not only the inner workings of the model but how the creation of such geographically explicit models helps with our understanding of such phenomena. This is then followed by discussion and conclusions concerning this particular modelling approach with respect to the study of urban phenomena.

## 2: Issues Involving Residential Segregation

People become geographically separated along many different lines and in different ways. There is segregation by gender, sexual orientation, age, income, language, colour, taste, comparative advantage, and accidents of historical location. Some segregation is organised, some is economically determined, some results from specialised communication systems, and some results from the interplay of individual choices that discriminate (Schelling, 1969). It is not just residential groups that segregate for segregation can take many other forms. Types of land-use such as residential, commercial, and agricultural are segregated in space. Types of businesses and industries are also segregated, often in clusters that indicate how their economic functionality relates to one another.

Segregation is all too clear in most urban areas, where there are clear clusters of economic groups and residential groups based on ethnicity or social class. Residential segregation has particular significance for cities as it can impact on the level of joblessness, out-of-wedlock birth, level of criminality, low educational achievement, income inequality and poverty traps (see Conejeros and Vargas, 2007). While we are able to quantify the degree of segregation within neighbourhoods (e.g. Reardon and O'Sullivan, 2004) which is much influenced by the choice of spatial units used (Bjornskau, 2005), this tells us little about the behaviour that leads to, or from particular outcomes, and without this knowledge, trying to prevent such a process or phenomena becomes challenging. For example, one might think that individuals must have strong preferences for these racially or economically homogeneous neighbourhoods to emerge. However, this is often not the case. Empirical evidence suggests that individuals do not have strong racial preferences but have rather mild preferences (e.g. Clark, 1991, Antonovics *et al.*, 2003). To find clear examples of this segregation process taking place is difficult, because it only becomes noticeable when it is clearly underway, and by then a detailed chronology becomes impossible to reconstruct (Batty *et al.*, 2004). To understand this behaviour, we have to examine how the process of individual choice leads to these outcomes. Schelling (1969) was one of the first to highlight the logic of how geographical segregation along racial lines can result from mild discriminatory choices by individuals.

Schelling presented his hypothesis using a two-dimensional checkerboard model and through the use of simple logic he illustrated how segregation could emerge (Schelling, 1971). He imagined this large checkerboard to be a city, with each square of the board representing a house or a lot, in which he placed equal numbers of two types of agents representing two groups in society. The types can be used to represent different social classes, racial groups, sexes etc. Initially these are placed at random across the board, with no more than one per square and a small number of squares are left vacant so that people can move. The rule of the model involves whether an agent decides to move or not. The agent decides to move from its square to an empty one if less than a specified percentage of its neighbours are of the same type as itself. The game progresses in a series of steps, where at each step, an agent is chosen and can decide whether or not to move. If the agent decides to move, the agent moves to the nearest vacant square that meets its demands (Schelling, 1971). As the simulation progresses, the two types of agents divide themselves up into sharply segregated groups. The model shows that segregation emerges through mild preferences to locate amongst like–demographic or economic activity groups where strict segregation emerges unknowingly. Subsequent researchers have endorsed his conclusions (see Clark, 1991). However his work has also received criticism; for example, Massey and Denton (1993) correctly point out that the 'residential-tipping' point model is not sufficient in itself as an explanation of segregation for many reasons. They comment that while it accurately captures the dynamic effects of prejudice, it accepts as a given the existence of racial discrimination. But what really matters is that individuals have preferences for both place and people (Bjornskau, 2005).

Unknowingly, Schelling was one of the pioneers in the field of ABM (Schelling, 2006). He emphasised the value of starting with rules of behaviour for individuals and using simulation to discover the implications for large scale aggregate outcomes through the interactions of these individuals. This key feature of the model arises because the decisions of any one individual can impact in unexpected and unanticipated ways upon the decisions of others. A group of individuals can be perfectly happy in a neighbourhood. Unexpectedly, an agent arrives to fill an empty space. The newcomer may tip the balance -'residential tipping' - so the agents who were previously content now decide to move. In turn, their moves may disrupt settled neighbourhoods elsewhere and so the effects percolate through the system. No single individual intends this to happen or even necessarily desires this overall outcome. They are only concerned about what happens in their own local area where they are content to integrate, but local interactions between them produce global segregation.

Not only is the model one of the best known agent-based models, it has additionally continued to inspire theory and research into the segregation phenomena (e.g. Clark, 1991; Bruch and Mare, 2005; Pancs and Vriend, 2007). For example, Bruch and Mare (2005) compared Schelling's model with stated preference data on residential choice for various race-ethnic groups (e.g. Asians, Hispanics, whites and blacks) within American cities. The preference data showed that most people were unwilling to live in neighbourhoods in which their own race-ethnic group is the minority. Of course, the world is more complicated than that described in the model. In reality, for example, not everyone has the ability to move (residential mobility) and people are not initially scattered randomly across the city. However, this does not undermine Schelling's central insight: marked segregation can arise from rather mild individual preferences for living amongst one's own kind.

## 2.1: Agent-Based Models of Segregation

As noted above many agent-based models have been inspired by Schelling's (1971) model or can be seen as extensions to its original insights and in this section, we will briefly explore some of these. Various neighbourhood shapes and sizes have been investigated to explore their impact on segregation outcomes. For example, Flache and Hegselmann (2001) have studied the Schelling model using a Voronoi partition of space which vary the structure and size of neighbourhoods, where they demonstrate that the model results are robust to such variations in grid structure. Laurie and Jaggi (2003) along with Fossett and Waren (2005) used variations of neighbourhood sizes (termed vision) and discovered that with larger von Neumann-like neighbourhoods and stronger preferences to be with like groups, more extreme patterns of segregation would emerge. O'Sullivan *et al.*, (2003) developed a model which considered both local Moore neighbourhoods (with eight surrounding cells) which they termed continuous and larger bounded neighbourhoods (e.g. five by five cells) within a regular lattice structure. Household agents consider their continuous neighbourhood as well as the aggregate information from the bounded neighbourhood that contains their residential location when deriving their satisfaction from a particular area. If an agent is on an edge cell of one of the bounded areas, it would take information from its continuous neighbourhood which may overlap more than one bounded area but only the aggregate information from the bounded area it falls within. Agents only move if they are dissatisfied, in which case the agent moves to an area where it is satisfied with both the local and bounded neighbourhood. The study showed that if the bounded neighbourhoods increased in size, the model takes longer to stabilise.

Others have extended the Schelling model to incorporate other factors into their models rather than just preferences for social/racial groups, such as the inclusion of preferences for neighbourhood status and housing quality, and differing levels of socio-economic inequality within and between ethnic populations (see Fossett and Senft, 2004). Benenson (1998) explored how a theoretical city evolved when agents have both economic and cultural preferences. Bruch (2006) explored the relationship between race and income, and how both interact to produce and maintain segregated neighbourhoods within Los Angeles. Within the model, agents were given a race and an income, and the model examined the probability of an agent moving into a neighbourhood of a given racial and economic composition.

Researchers from Tel Aviv University have been particularly active in using ABM to simulate segregation and residential dynamics. They have investigated residential dynamics using agent-based models from abstract systems to real-world examples (see Benenson, 1998; Portugali, 2000; Benenson *et al.*, 2002; Omer, 2005). Omer (2005) extended the Schelling model to include a further hierarchical level; that is, the agents' ethnic identities are organised in a two-level hierarchy where each agent belongs to an ethnic group and a subgroup. For example, the British Asian community is multi-differentiated in terms of nationality, country of origin, religion, caste, class and language. Extending the Schelling model to include an additional hierarchical level allows for further research dealing with the role of ethnic preferences on residential choice. Like the Schelling model, it is based on an abstract city using cells to represent houses. An agent's decision to move is dependent on the properties of the agents living in its current as well as potential destination neighbourhoods.

Of special interest here is the study of fine scale residential segregation using individual census records and GIS data representing streets and buildings (see Benenson and Omer, 2003). Benenson *et al.* (2002, 2003) have used this kind of detailed dataset to simulate ethnic residential dynamics between 1955-1995 in the Yaffo area of Tel Aviv. The model itself consists of two interacting layers, one layer representing mobile agents comprising of three cultural groups - Jews, Arab Muslims, and Arab Christians, located on a physical environment layer representing streets and buildings. Each house is converted into a Voronoi polygon and the agents' residential behaviour is affected by the ethnic composition of the neighbourhood defined using these polygons. A neighbour is a Voronoi polygon that has a common boundary (roads act as barriers between these neighbourhoods) while another difference between this and other segregation models is that there are more than two types of agents interacting within the system.

The examples presented in this section can be viewed on a continuum between abstract demonstrations to real-world applications. Each one brings something new to the basic insights Schelling first presented. There are those that apply the Schelling model to empirical data (e.g. Bruch and Mare, 2005), those that explore the effect of differing neighbourhood sizes (e.g. O'Sullivan *et al.*, 2003) or shapes (e.g. Flache and Hegselmann, 2001), those that extend the Schelling model to incorporate subgroups (e.g. Omer, 2005) and those that introduce other determinants of segregation (e.g. Fossett and Senft, 2004). Benenson *et al.*'s (2002) work is the exception in that it directly relates to actual places and includes more than two types of agent.

As noted in the introduction, ABM and simulation has long been dependent on rectangular grids to represent both spatial and social relationships. While this has been productive for many kinds of simulation, researchers have started using irregular spaces (see Semboloni, 2000; Shi and Pang, 2000; O'Sullivan, 2001; Flache and Hegselmann, 2001; Benenson *et al.*, 2002), and have discovered that many models are sensitive to variations in the structure and size of neighbourhoods between locations in the grid (e.g. O'Sullivan, 2001). This is also seen by those developing Schelling-like models. The majority of the Schelling models described above use a regular cellular partition to represent space. Each cell is often used to represent a single home, with one agent being allowed to occupy the cell at any one time (e.g. O'Sullivan *et al.*, 2003; Omer, 2005). However, it is argued that this is unrealistic especially within cities; for example, within a block of flats there can be numerous people but their geographical footprint would be the same and would be missed by restricting one agent to one cell. Most of these models and to a similar extent empirical analysis exploring segregation, employ a featureless plain, playing little attention to physical barriers. Noonan (2005) showed empirically how physical barriers (such as parks, railroads, major roads and industrial corridors) have impacts on neighbourhoods. For example, gated communities present a sharp example of the use of physical features to insulate against neighbours (see Helsley and Strange, 1999). However, while including basic geometrical properties, all these models do not allow for overlap between areas. The remainder of this paper will present how individual entities can be created and located within space where movement is not restricted by cells as the model contains no cells. Furthermore, more than one agent can be located in the same area, and clusters of residential groups can bridge different areas.

## 3: A Vector Based Geographically Explicit Segregation Model

As discussed above, there are many types of segregation which are a product of many factors. The model presented in this paper only explores one such hypothesis, that of Schelling's (1971, 1978) original model where with agents of mild tastes and preferences to locate amongst like social-groups, segregation will emerge. The model is therefore uncluttered with additional variables that might affect segregation such as how economic factors may contribute to racial segregation based on systematic income differences across groups as well as price and quality of life arising from lot size and other amenities. The purpose of this model is to simply extend this well known model so we can explore the impact of space and geometry on such a process; it is a pedagogic demonstration to simply articulate a way of thinking about modelling the built environment in the particular context of segregation. The following section will outline the basic model, while further details of the model, including the source code, a complete description and animations of simulation runs can be found at [www.casa.ucl.ac.uk/abm/segregation](www.casa.ucl.ac.uk/abm/segregation). This supplementary material is provided to aid in understanding the model, to demonstrate how patterns of segregation develop over time, and to allow for replication and extension (i.e. the inclusion of other variables) if desired, all of which are advocated by Axelrod (2007).

The model itself is loosely coupled to geographic information systems (GIS) especially the vector data structure, written in Java and extends a number of basic operating classes from the RepastJ library, an open source Java based agent-based toolkit[2] (Repast, 2008). Within the model, Repast is primarily used for its display, scheduling, the import of GIS vector data (in the form of ESRI shapefiles), along with its recording change classes. The model additionally utilizes other Java based libraries, namely the Java Topology Suite (JTS, 2008) which provides general 2D spatial analysis functions such as professionally developed line intersection and buffering algorithms, and OpenMap (2008) which provides a simple GIS display with pan and zoom, and query functions with respect to GIS layers.

Typical of most ABM, the development of the model involved an iterative process where model verification was based on many iterations of the system. Each iteration extends the basic model providing greater realism and functionality. Unit testing was undertaken after every adjustment/iteration to the programming code. This unit testing allowed for confidence to be gained in the model, specifically in terms of model processes taking place at the right time, and each process occurring in the manner in which it was intended. This permitted the identification of unexpected outcomes of the model itself as opposed to errors ('bugs') in the code (Gilbert and Terna, 2000).

---

[2] A toolkit is a simulation / modelling system that provides a conceptual framework for organising and designing agent-based models. It provides appropriate libraries (a collection of programming classes grouped together, termed packages i.e. classes with similar purpose) of software functionality that include pre-defined routines / functions specifically designed for ABM. Toolkits provide reliable templates for the design, implementation and visualisation of agent-based models, allowing modellers to focus on research (i.e. building models), rather than building fundamental tools necessary to run a computer simulation (Castle and Crooks 2006).

In particular, the use of toolkits can reduce the burden modellers face programming parts of a simulation that are not content-specific (e.g. a Graphical User Interface, GUI, data import-export, visualisation / display of the model). Toolkits also increase the reliability and efficiency of the model, because complex parts have been created and optimised by professional developers, as standardised simulation / modelling functions. However, there are limitations of using simulation / modelling systems to develop agent-based models, for example: a substantial amount of effort is required to understand how to design and implement a model in some toolkits (Castle and Crooks 2006).

Within the model, we consider agents as virtual households with the ability to search the virtual world and make residential choices. These agents possess an ethnic status which we denote for example as red and blue with these labels of course arbitrary. These households have preferences for co-ethnic contact specified in terms of the percentage of co-ethnic households found in their ‘neighbourhood’ in which the household lives or to where it is considering moving. Preferences can be the same or different between different ethnic groups. Unlike many models exploring segregation (e.g. Laurie and Jaggi 2003; Fossett and Waren, 2005), these households are not restricted to discrete cells and can move to areas which are already occupied by other agents. The boundaries of the virtual world act as physical boundaries. Instead of a ‘wrapping-around’ to meet each other, often called an edgeless torus’ (e.g. the Laurie and Jaggi (2003) model), the boundaries or ‘edges’ are analogous to those of ‘real’ urban areas.

Translating GIS methods into agents and their environment, the model is comprised of two vector layers – the urban environment which is represented as a series of polygons created directly from the shapefile, and agents which are represented as points. It is the information held within fields of the environment layer that is used to create the point agents. The distribution of the types of point agents (representing ethnic groups, say) as observed through aggregate census population counts form the initial starting conditions for the model. For example, Figure 1A represents four wards in the City of London each with their own attribute information stored in a data table where each row relates to a specific ward (e.g. ward 1 has a population of ten red, five blue, four green and two white agents). The model reads this data and creates an environment polygon for each ward and for the desired agent population based on data held in the fields as shown in Figure 1B. Note that the underlying colour of the polygon (ward) always represents the predominant social group in the area (accomplished by counting the number of agents of different types within each polygon). The agents are initially randomly placed within each polygon. This provides a city landscape that is integrated at initialisation. However these agents could be placed in precise locations if they were known (see Crooks, 2007 for further details). The basic model is designed to work on many different geographical scales (e.g. boroughs, wards, output areas, and OS MasterMap TOIDs®) without the need for its reconfiguration as we indicate in Figure 2. This was considered important as most socio-economic data for example, census and geo-demographic data comes in this format. This functionality was created so that the model could be easily replicated in other areas in the quest to allow the modeller to see if the same rules can be applied to different areas and at different scales.

The ability to represent the urban system as a series of spatial objects – points, lines and polygons each with a spatial reference describing the location of the object rather than just as a series of cells, leads to conceptual problems in defining neighbourhoods. Secondly it makes definitions of the ways in which agents move and search their environment difficult. To overcome these problems, the model relies on a series of spatial analysis operations specifically, buffering, union, line intersection and point(s)-in-polygon analysis utilizing the JTS library. It is to these we now turn.

| Ward | No. Red | No. Blue | No. Green | No. White | Other info |
|---|---|---|---|---|---|
| 1 | 10 | 5 | 4 | 2 | |
| 2 | 5 | 10 | 3 | 4 | |
| 3 | 2 | 3 | 5 | 3 | |
| 4 | 2 | 3 | 4 | 2 | |

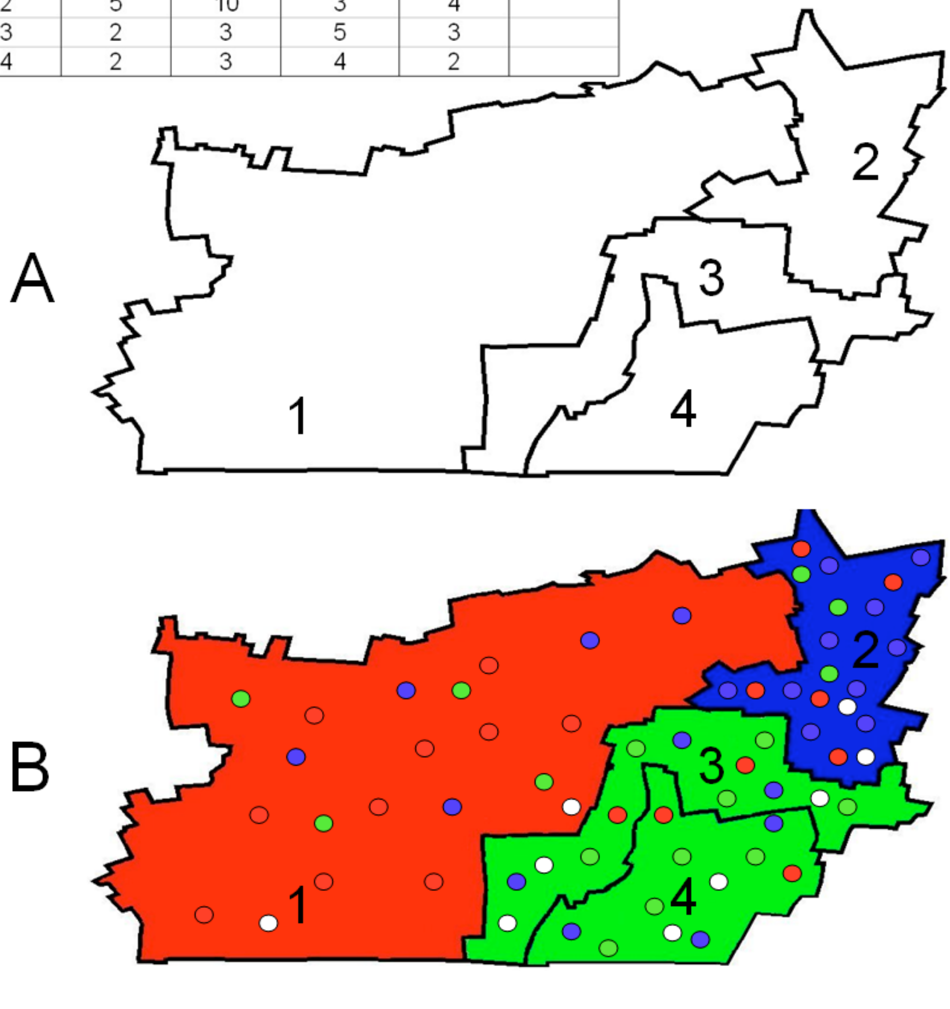


**Figure 1:** Reading in the data and creating the agents

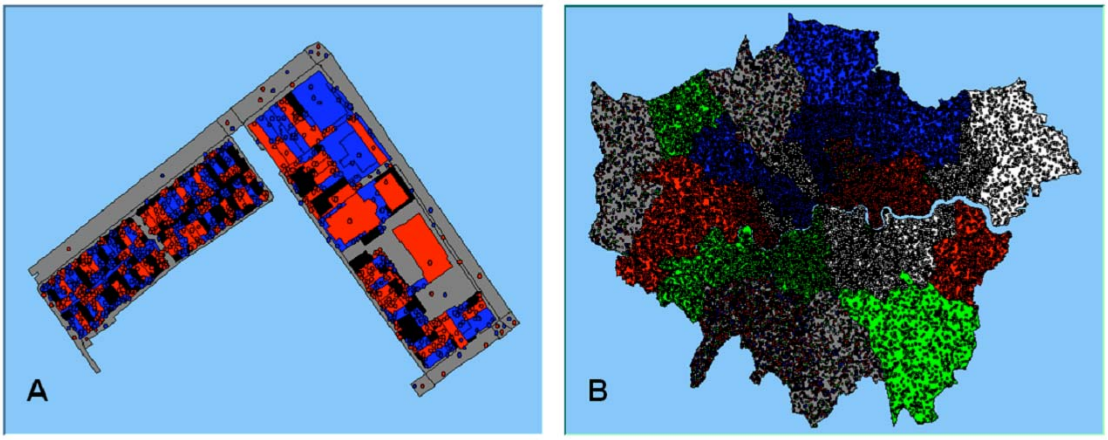


**Figure 2:** Spatial representation within the model
A: A street section with building footprints. B: London composed of boroughs.
Agents are shown as dots.

Unlike that of cellular space models where neighbourhoods are often calculated using Moore or von Neumann neighbourhoods or variations of these (Batty 2005b), representing agents as points mean different tools are needed to calculate neighbourhoods, specifically when incorporating physical boundaries (e.g. rivers and motorways) into the modelling process. For the point agents, buffers are created to calculate neighbourhoods. This involves the creation of a circular region around a point. The radius of the circle is defined by the analyst using Euclidian distance. Buffers are often used in GIS to reflect notions of accessibility or proximity to geographical features.

Figure 3A highlights how a geographical feature (such as a river) can be incorporated into the model when calculating neighbourhoods for point agents. Within Figure 3A, the black circle represents the point agent of interest. This agent wants to know which agents are within a specified distance of itself and in the same geographical area. A buffer is created at this specified distance based on the centroid of the agent. However, in this case, the buffer crosses the river. Therefore agents on the other side of the river (yellow squares) are not neighbours as there is no way for them to move directly to the agent; however, they are within the buffered region (green line). Those agents (red squares) which are on the same side of the river as the agent and are within the agents' defined buffer (red line) are classed as neighbours, and any agents outside this area would not be classed as neighbours. This creation of buffers also has the advantage of calculating local statistics such as population density of small areas. However, if there was a bridge connecting the two areas, say by loading a new shapefile into the simulation, the agents on the other side of the river would be considered neighbours as demonstrated in Figure 3B.

Agent-based models create worlds populated by agents where these agents are free to explore this world. However, such worlds often have boundaries. Within a cellular world, this is based on a matrix of cells, for example, a 10x10 regular lattice of cells, say. However, as one moves to a representation of space using irregular cells, this is no longer the case. One has to define the boundary of this world. To accomplish this task, the model uses the spatial analysis operation known as union. Simply stated, union combines the polygons and returns one extent of the whole area. This is then used as the boundary for the model which retains the original geometry and restricts agents' movement to within it.

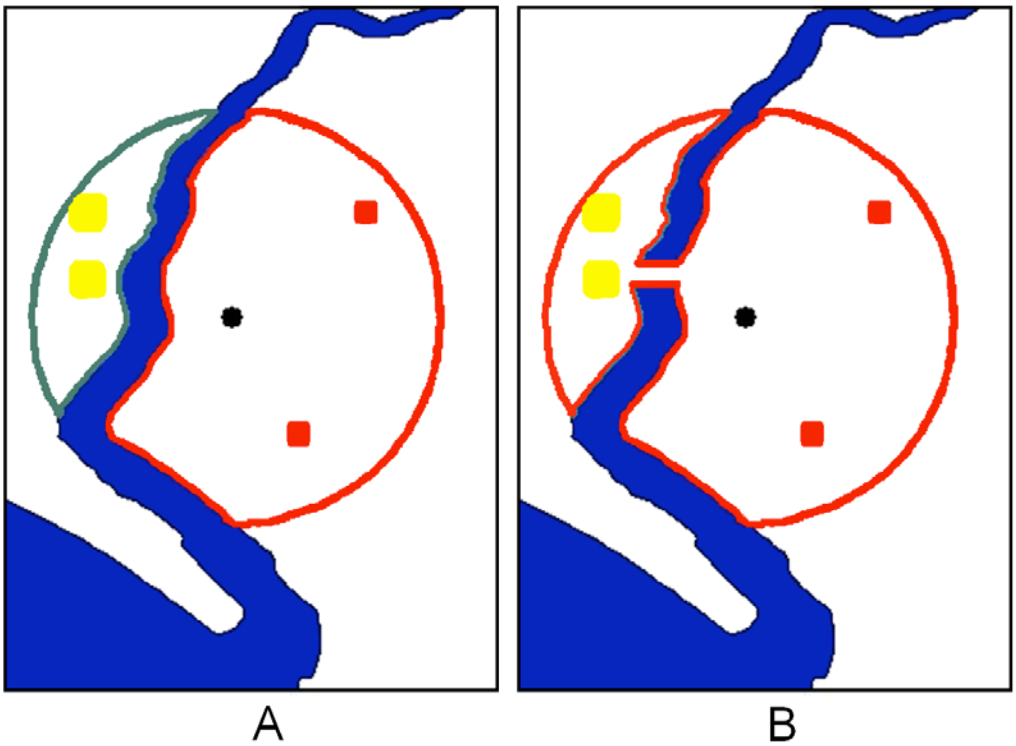


**Figure 3:** A: Defining neighbourhoods with the inclusion of geographical features (constrained buffer). B: Defining neighbourhoods where the two areas are connected by a bridge.

As each agent class is represented as a separate layer, one needs to relate objects in one layer to objects in another. This is achieved through a point-in-polygon analysis. This allows the model to determine whether a given point lies inside a specific polygon. The ability to carry out point-in-polygon operations, with as for example, the number of point agents that are contained within a polygon, allows the model to be generative. In this sense, entities at higher levels of geographic representation such as population counts are derived from the bottom-up, through interactive dynamics of collections of animate objects observed at the micro-scale.

The above GIS operations allow an agent-based model to be created where space and geometrical relationships are explicitly incorporated into the simulation. Each type of agent knows its position and can use the operations such as buffering and point-in-polygon to find out more about its neighbourhood as we show in the basic model structure in Figure 4. Once the environment and the agents have been created, each agent users its neighbourhood function to query the surrounding neighbours, calculating if it is currently satisfied with its current neighbourhood environment taking into account physical features of the urban environment. Figure 4 highlights this process, where an agent is selected at random and it 'evaluates' the ethnic mix in its immediate neighbourhood. If the agent is satisfied based on its preferences, it does nothing. On the other hand if it is dissatisfied with its current neighbourhood, the agent moves to the nearest location where its preferences are met.

Before discussing this movement in more detail, the dynamics in the model need to be briefly addressed. As with other segregation models, the time frame within the model is purely hypothetical but could be considered as yearly intervals. At each time step (iteration) of the model, all the agents are given the option to move if they are dissatisfied with their current neighbourhood configuration. The order in which an agent is chosen is random but if it does decide to move, the movement process involves two stages which we show in Figure 5. First, the agent randomly searches its local area for a given number of moves, each time moving to a new location, calculating the neighbourhood composition using the buffering mechanism. If the agent is still dissatisfied after a given number of random movements, it moves to the nearest neighbour of the same type based on Euclidian distance from its initial location. Once the agent has moved to its nearest agent of the same type, it locally searches this new area for a neighbourhood composition that satisfies its needs. If the agent cannot be satisfied, it moves to the next nearest agent and so on until all the area has been searched, if the agent is not satisfied then, it is removed from the system. Once all the agents have had the option to move, the model advances one iteration and again all agents who are dissatisfied with their neighbourhood have the option to move. This process continues until all the agents are satisfied with their current neighbourhood configuration or the model is stopped by the user.

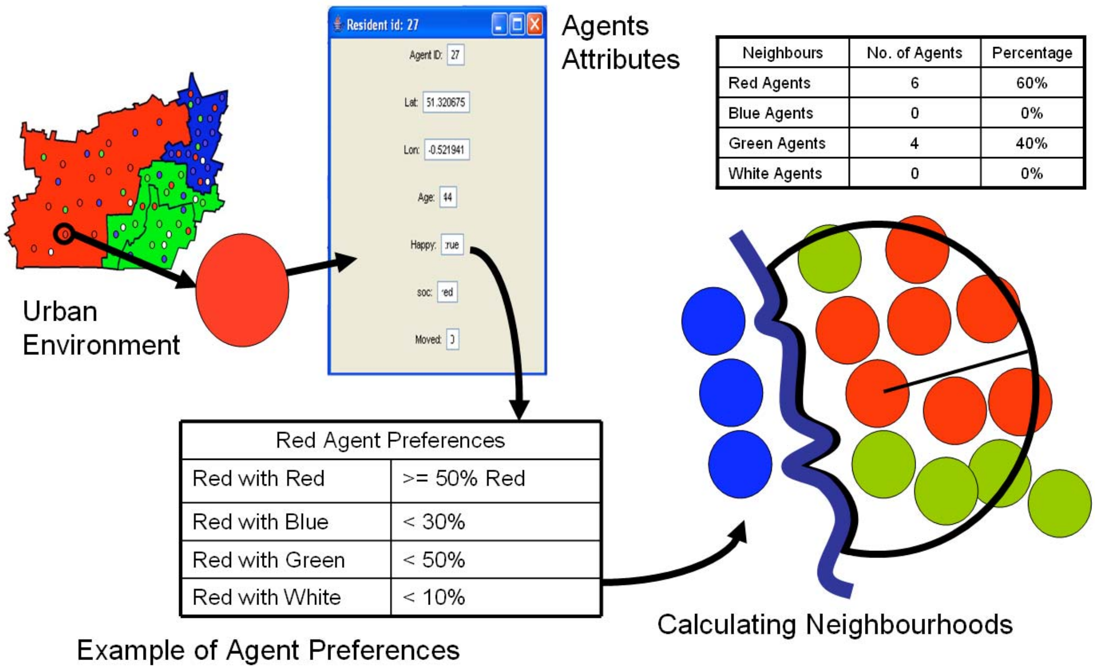


**Figure 4:** Basic model structure

Finally this model is highly visual. In Figure 6, we highlight the graphical user interface (GUI) to the model which provides an essential lens for viewing the operation of the model and judging its results. Clockwise from the top left is the control bar for the simulation, the GIS display utilizing OpenMap which shows the agents and the urban environment (i.e. wards in the City of London), graphs for aggregate outputs, a legend for interpretation of the GIS interface, model output in the form of text, and the model parameters all of which utilizing and extending Repasts inbuilt functions for GUI and data analysis. Furthermore the model is loosely coupled to ArcGIS in terms of its inputs and some outputs in terms of shapefiles through the linkage between Repast and GeoTools[3] Java libraries.

[3] http://geotools.codehaus.org/

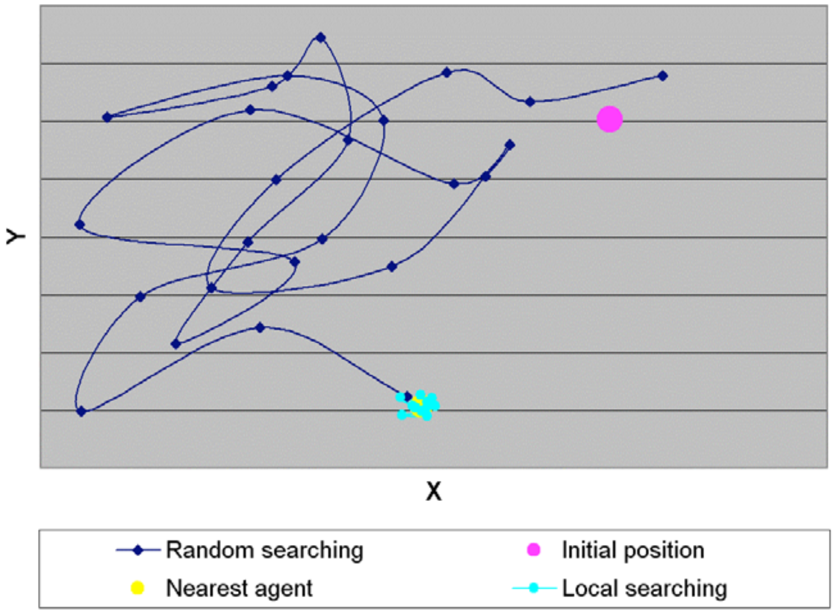


**Figure 5:** An example of searching for one point agent. From the initial location, this shows the random search path before moving to its nearest neighbour and carrying out a local search.

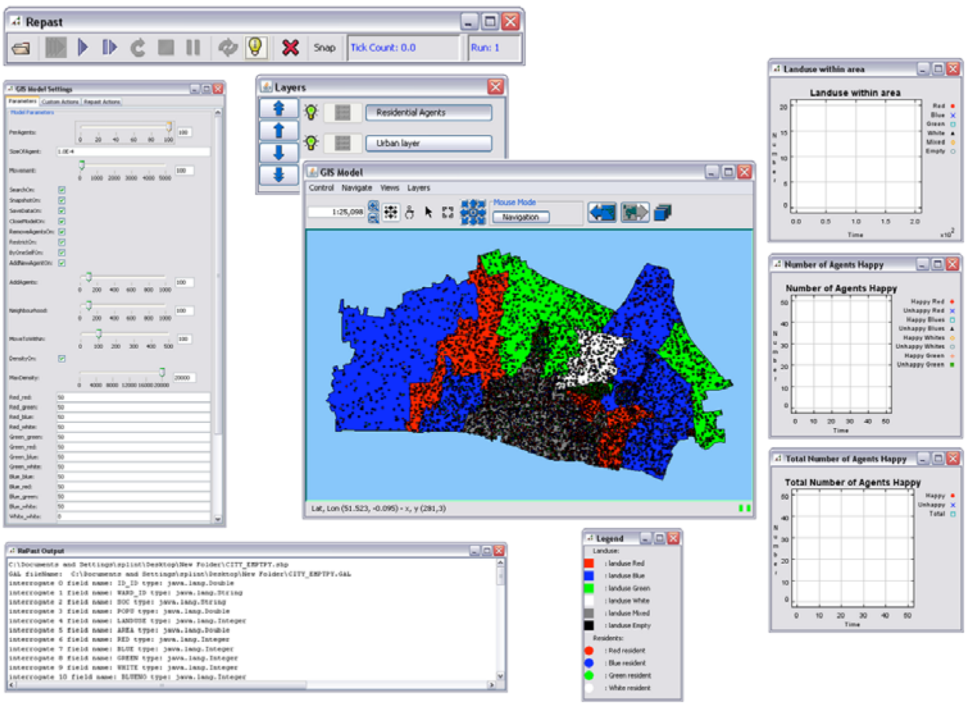


**Figure 6:** The Segregation Model User Interface. The pattern of segregation dimensioned to the geography of wards in the City of London.

## 4: Results from Selected Experiments

The following section will briefly present several simulation results. However, before presenting these experiments, a caveat is needed. While numerous experiments, variations or extensions of the model are possible, the following subsections only highlight how certain assumptions affect the outcome of the model and how changes can easily be made to the basic principles of segregation. First we will present how different degrees of segregation arise from different preferences of neighbourhood composition with two types of agents. Then we explore how the pattern of segregation is affected by different neighbourhood sizes, the role of geographical features, the result of different types of agents with a small minority population and lastly the result of the addition and removal of agents in the system. Not only do these demonstrate the structure of the basic model but these emphasise the rationale to why such features where included into the model.

Each simulation was run multiple times and the results shown below represent average outcomes. As with Schelling's original model, the simulations result in segregation emerging but the precise spatial pattern varies with different model runs. Not only do these simulations allow one to see the sensitivities of the model through its inner workings but also how the use of vector-based geometries can be incorporated into the simulation process. The data files and the model source code for each simulation can be found at www.casa.ucl.ac.uk/abm/segregation which allows for replication of the simulation, thus aiding verification of the model. Furthermore the website provides a medium for the visualisation of the dynamics within the model and the sharing of knowledge by providing detailed documentation of the model implementation, a modelling strategy advocated by Axelrod (2007). Many model details are hard to easily share or portray in the confines of a publication and readers are urged to consult the site for more details.

### 4.1: The Role of Preferences

As with Schelling's (1971) original model, agents only have preferences for their own group and it is this preference that causes agents to seek out different areas in the city. Clarke (1991) provides some empirical evidence to support Schelling's abstract formulations based on the fact that neither Black nor White households in several

American cities would relocate into areas where they are the minority. However in his work, preferences for specific composition of a neighbourhood varied among cities. This section will therefore explore how the degree of segregation changes due to different preferences and explore how the preferences of individuals for their own group influence the degree of segregation seen within an area.

The only model parameters that change within the model were the agents' preferences for the percentage of their same type to be located within their neighbourhood. Agents are satisfied in an area if their preferences are achieved. For example, a red agent may have a preference parameter for 40% of their neighbours to be of the same type as itself. Therefore if 40% or more of its neighbours are of the say type as itself, it is satisfied with its current location. When calculating this satisfaction, the agents do not count themselves during the process. Within these simulations, the world the agents occupy is a 1.5km by 1.5km square polygon which could be considered as representing a cityscape. One could imagine this as the checkerboard that Schelling originally used. However, neither agents' neighbourhoods nor their movement was restricted to a cell-based environment and multiple agents can occupy one area. As with Schelling's original model, we have equal numbers of two types of agents, 2000 of each colour placed randomly within the area.

Figure 7 highlights the typical patterns of segregation that emerge from different preference parameters for neighbourhood composition. As the percentage of neighbours of their same type increases, the pattern of segregation becomes more noticeable. It is only when preferences rise above 80%, that agents are forced to leave the system as a result of their preferences being unable to be matched as indicated in Table 1. Not all the agents are removed for as the system becomes less populated, the number of agents in different neighbourhoods changes releasing spaces. Where agents have been removed from the system, this removal only happens in the first iteration and for the first agents that move. These agents are unable to find a suitable neighbourhood due to their initial random placement and mixed neighbourhoods at the start of the simulation. As these agents are removed, the area becomes less populated and the resulting agents can find neighbourhoods where their preferences can be satisfied. While it is possible to add these removed agents back into the system at the end of the simulation, it was felt to be simpler to leave them out, as this reflects the idea that as an area changes, residential groups are actually excluded from those areas.

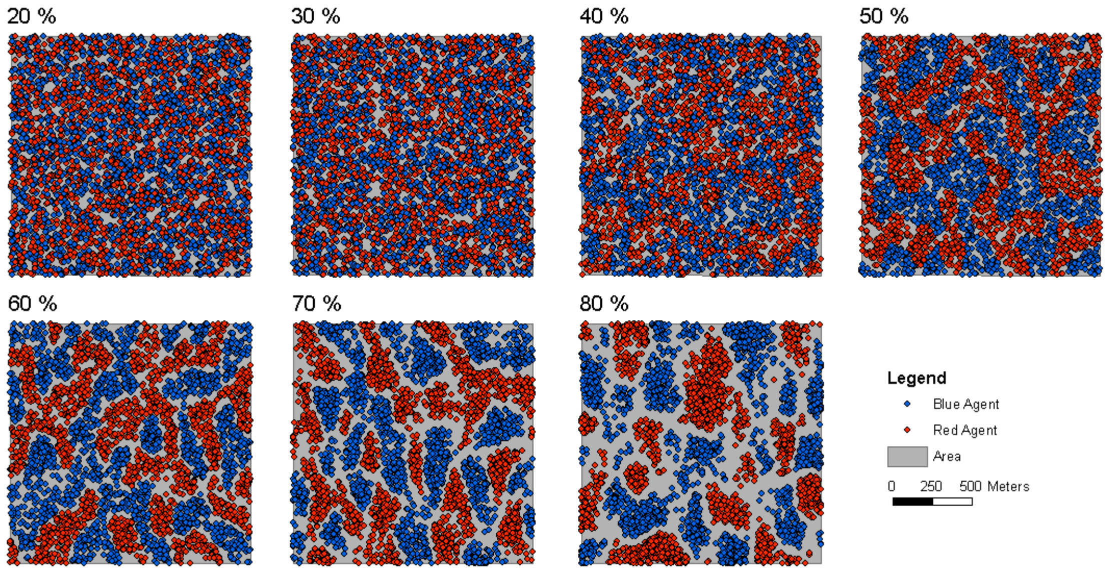


**Figure 7:** Typical patterns of segregation with different percentage preferences for neighbourhood composition.

Table 1 also highlights that by increasing the percentage of neighbours of the same type within the agents' neighbourhood, more agents are forced to move at least once during the course of a simulation. For example when preferences are low (e.g. >= 20%), little movement occurs. However, as the preference for a minimum neighbourhood increases, so does the total number of agents that move (e.g. >= 40%) and the resulting pattern of segregation increases as highlighted in Figure 7. The frequency of movement is not often analysed in previous models exploring segregation and thus it is interesting to note how often agents move throughout the course of a simulation. Agents only move to areas which they are satisfied with, and therefore for an agent to move more than once suggests, that 'residential-tipping' is occurring.

Although patterns can be deceiving and it is useful to have some measure of segregation, one possible measure is the average proportion of neighbours of like or opposite colour. By counting the total number of neighbours of different types for each of the agents remaining when all are satisfied with their neighbourhood, a greater understanding of the degree of segregation can be gained. At the same time, this allows for testing if neighbourhood and preference functions in the model are working correctly.

**Table 1:** Comparison of neighbourhood preferences and model runs.

| Preference | Number of Agents Remaining when all Satisified | | | | | | Iterations Untill all Agents are Satisfied | | Moves | | | | | | | | | | Total Number of Agents that Moved | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Total | | Red | | Blue | | | | 0 | | 1 | | 2 | | 3 | | 4 | | | |
| | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev |
| 20 | 4000.0 | 0.0 | 2000.0 | 0.0 | 2000.0 | 0.0 | 1.3 | 0.7 | 3996.7 | 3.3 | 3.3 | 3.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 3.3 | 3.3 |
| 30 | 4000.0 | 0.0 | 2000.0 | 0.0 | 2000.0 | 0.0 | 2.8 | 0.6 | 3937.8 | 14.0 | 59.6 | 12.0 | 2.6 | 2.4 | 0.0 | 0.0 | 0.0 | 0.0 | 62.2 | 14.0 |
| 40 | 4000.0 | 0.0 | 2000.0 | 0.0 | 2000.0 | 0.0 | 10.2 | 1.9 | 3091.2 | 36.3 | 743.6 | 25.9 | 147.8 | 20.8 | 16.6 | 5.6 | 0.8 | 0.8 | 908.8 | 36.3 |
| 50 | 4000.0 | 0.0 | 2000.0 | 0.0 | 2000.0 | 0.0 | 6.1 | 0.7 | 2129.5 | 29.3 | 1626.7 | 25.8 | 230.6 | 19.9 | 12.7 | 3.9 | 0.5 | 1.0 | 1870.5 | 29.3 |
| 60 | 4000.0 | 0.0 | 2000.0 | 0.0 | 2000.0 | 0.0 | 4.0 | 0.8 | 1685.6 | 41.1 | 2232.0 | 47.0 | 81.7 | 17.2 | 0.7 | 0.7 | 0.0 | 0.0 | 2314.4 | 41.1 |
| 70 | 4000.0 | 0.0 | 2000.0 | 0.0 | 2000.0 | 0.0 | 3.1 | 0.6 | 1212.1 | 53.6 | 2752.6 | 52.9 | 35.3 | 8.6 | 0.0 | 0.0 | 0.0 | 0.0 | 2787.9 | 53.6 |
| 80 | 3999.9 | 0.3 | 1999.9 | 0.3 | 2000.0 | 0.0 | 2.6 | 0.5 | 796.7 | 38.1 | 3181.5 | 39.6 | 20.5 | 6.3 | 0.2 | 0.6 | 0.0 | 0.0 | 3202.2 | 36.9 |

Table 2 presents the average neighbourhood composition in terms of the percentage and number of agents at the end of each model run when all the agents have their preferences satisfied. As one would expect as the agents preference for a certain composition of a neighbourhood increases, the degree to which neighbourhoods are segregated (percentage composition and number of the same type) also increases (e.g. from 40% onwards). The most noticeable variation is at 50% where the degree to which neighbourhoods are segregated rises the most.

**Table 2:** Comparison of mean percentages of neighbourhood compositions for different preferences when all agents are satisfied.

| Preference | Blue Agents | | | | Red Agents | | | |
|---|---|---|---|---|---|---|---|---|
| | Mean % of Neighbours | | Mean No. of Neighbours | | Mean % of Neighbours | | Mean No. of Neighbours | |
| | Blue | Red | Blue | Red | Blue | Red | Blue | Red |
| Average Starting Conditions | 51.0 | 49.0 | 27.3 | 26.4 | 49.1 | 50.9 | 26.3 | 27.2 |
| 20% | 51.0 | 49.0 | 27.3 | 26.4 | 49.1 | 50.9 | 26.3 | 27.2 |
| 30% | 51.1 | 48.9 | 27.3 | 26.5 | 48.8 | 51.2 | 26.4 | 27.5 |
| 40% | 55.5 | 44.5 | 30.0 | 25.3 | 44.6 | 55.4 | 24.7 | 29.5 |
| 50% | 67.0 | 33.0 | 36.3 | 20.4 | 33.0 | 67.0 | 18.6 | 34.6 |
| 60% | 77.6 | 22.4 | 42.5 | 15.7 | 22.4 | 77.6 | 12.8 | 39.8 |
| 70% | 87.0 | 13.0 | 52.7 | 12.5 | 13.0 | 87.0 | 7.8 | 48.4 |
| 80% | 93.2 | 6.8 | 76.7 | 11.1 | 6.2 | 93.8 | 3.9 | 70.4 |

## 4.2: The Effect of Different Neighbourhood Sizes

Neighbourhoods mean different things to different people. Some may perceive a neighbourhood as houses that are directly attached to their home (e.g. Benenson *et al.*,

(2002) model), while others may consider a street, or a collection of streets as their neighbourhood. Schelling (1971) presented two models with regard to neighbourhoods: one based on their immediate eight surrounding cells (Moore neighbourhood) and a second on a larger area – his 'bounded neighbourhood' model.

As the model does not operate on cellular space, neighbourhoods are not calculated using the traditional Moore or von Neumann neighbourhoods or variations of these. As already stated, neighbourhoods within the model are calculated using a buffer at a specified radius around the agent. To test the influence of neighbourhood size (which some class as vision e.g. Laurie and Jaggi, 2003; Fossett and Waren, 2005) on the resulting pattern of segregation that emerges, various neighbourhood sizes were tested ranging from a radius of 50m to 1000m. All the other parameters within the simulations were kept the same: 4000 agents were randomly placed, 2000 of each colour within a 1.5km$^2$ area. All agents desire to be in a neighbourhood where 50% or more of their neighbours are of the same type. Increasing the neighbourhood size could be considered as a way of exploring both of the models that Schelling (1971) presented. As the smallest neighbourhood would only encompass the agents immediate neighbours, if an agent was dissatisfied with the area, this would reflect the composition of the agent's immediate neighbours. For larger neighbourhoods, agents would consider larger areas with the agent not considering its immediate neighbours per se but its overall neighbourhood composition.

Typical patterns of segregation resulting from different neighbourhood sizes are presented in Figure 8 which clearly shows the influence of neighbourhood size on the outcome of the pattern of segregation. Smaller neighbourhood sizes result in small segregated areas and larger neighbourhoods result in larger segregated areas in proportion to the size of the buffer used. Another feature of the model is that at the boundary of these neighbourhoods, agents appear to be more clustered than those in the centre of the neighbourhood.

From an examination of the summary statistics from these model runs shown in Table 3, the total number of moves agents made during a simulation, while being dependent upon the initial neighbourhood configuration, increases with neighbourhood size. This is especially the case for the number of agents that do not move during the course of the simulation and those agents that move more than once. As agents only move to areas in which they are satisfied, for agents to move more than once, their neighbourhood would have to have changed.

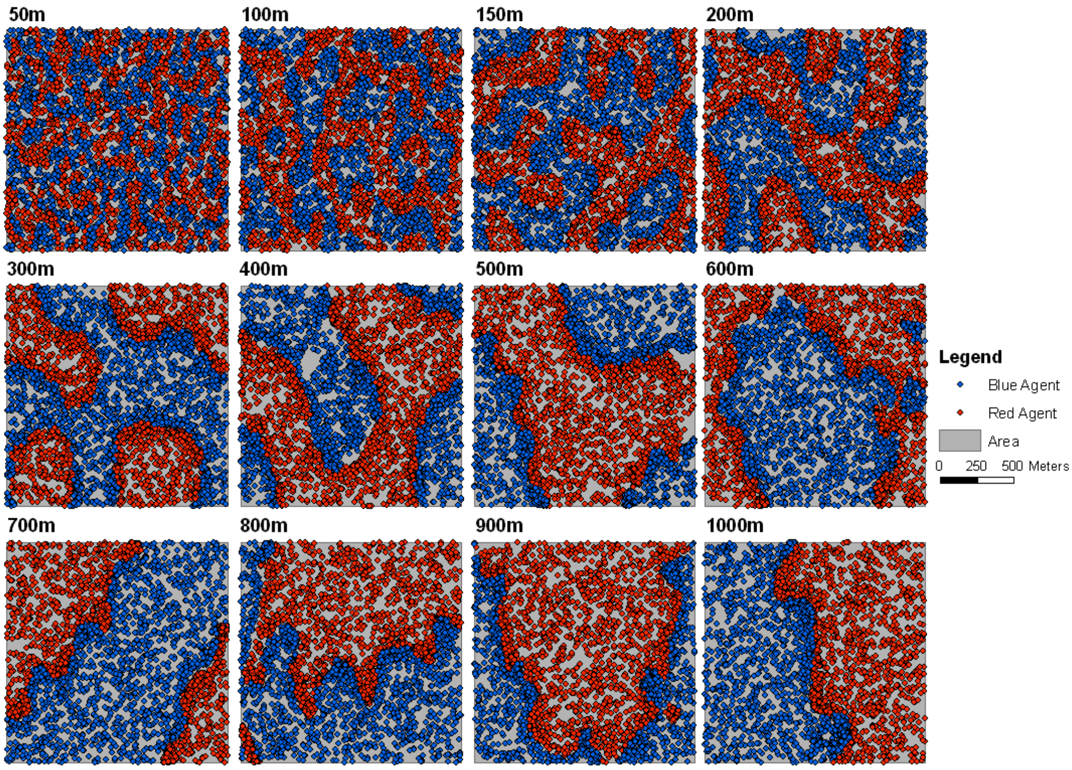


**Figure 8:** Example patterns of segregation when all agents are satisfied for different neighbourhood sizes.

**Table 3:** Comparison of neighbourhood size, number of iterations and number of moves until all agents are satisfied.

| Neighbourhood Size | Iterations Until all Agents are Satisfied | | Number of Moves | | | | | | | | | | | | | | Total Number of Moves for Agents | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | | |
| | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD | Mean | SD |
| 50m | 5.3 | 0.8 | 2358 | 42 | 1514 | 42 | 126 | 19 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1642 | 42 |
| 100m | 6.1 | 0.7 | 2130 | 29 | 1627 | 26 | 231 | 20 | 13 | 4 | 1 | 1 | 0 | 0 | 0 | 0 | 1871 | 29 |
| 150m | 6.6 | 1.6 | 2055 | 53 | 1620 | 30 | 303 | 31 | 21 | 6 | 1 | 1 | 0 | 0 | 0 | 0 | 1945 | 53 |
| 200m | 6.4 | 1.0 | 2012 | 32 | 1589 | 41 | 378 | 30 | 30 | 9 | 1 | 1 | 0 | 0 | 0 | 0 | 1998 | 46 |
| 300m | 5.9 | 0.9 | 1967 | 40 | 1543 | 50 | 442 | 60 | 46 | 17 | 2 | 3 | 0 | 0 | 0 | 0 | 2033 | 40 |
| 400m | 5.8 | 0.9 | 1935 | 73 | 1425 | 92 | 556 | 92 | 75 | 50 | 9 | 13 | 0 | 0 | 0 | 0 | 2065 | 73 |
| 500m | 5.8 | 0.8 | 1936 | 67 | 1420 | 74 | 546 | 72 | 91 | 43 | 7 | 9 | 0 | 0 | 0 | 0 | 2064 | 67 |
| 600m | 5.8 | 2.1 | 1919 | 82 | 1409 | 155 | 552 | 86 | 105 | 98 | 13 | 24 | 2 | 5 | 0 | 1 | 2081 | 82 |
| 700m | 5.8 | 1.5 | 1932 | 89 | 1416 | 143 | 544 | 111 | 95 | 94 | 13 | 27 | 1 | 1 | 0 | 0 | 2068 | 89 |
| 800m | 6 | 1.6 | 1918 | 50 | 1422 | 248 | 577 | 219 | 78 | 74 | 5 | 9 | 0 | 0 | 0 | 0 | 2082 | 50 |
| 900m | 5.6 | 1.5 | 1924 | 78 | 1523 | 148 | 496 | 133 | 55 | 50 | 2 | 2 | 0 | 0 | 0 | 0 | 2076 | 78 |
| 1000m | 5 | 1.2 | 1931 | 75 | 1638 | 191 | 410 | 148 | 21 | 10 | 1 | 2 | 0 | 0 | 0 | 0 | 2069 | 75 |

## 4.3: The Impact of Geographical Features

Areas within cities are bounded by features such as highways, railway lines, rivers, lakes, and parks which can act as boundaries between residential groups. For example, Rabin (1987) observed that highways formed effective boundaries between the predominant white and minority groups in residential areas in most U.S. cities. The model presented in this paper was designed to explore the effect these features have on the outcome of a particular simulation. Thus the segregation model is not only capable of exploring segregation but also examining the effect geographical features have on the pattern of segregation that emerges (i.e. the ability of geographical features to separate neighbourhoods). The following section will therefore demonstrate how neighbourhoods can be influenced by the geometrical features of the urban environment. To achieve this, the segregation model will be compared to a variation that does not include geographical features (geometry) when neighbourhoods are being calculated.

An arbitrary area was used to represent the urban space as shown in Figure 9. The grey area represents locations which agents can be inhabit while the blue area represents areas where the agents cannot be located and could be considered void areas such as water features. The region was manipulated so that there are geographical areas where agents can be located directly opposite each other but are separated by void areas which act as barriers in the neighbourhood calculation in the basic segregation model where buffers are created and constrained by geographical features as we noted in Section 3 and highlight in Figure 9. They would only be included in the variation of the model where geometry is not considered when calculating neighbourhoods (i.e. the buffer is not constrained by geographical features/areas and can cross this void space as we show in Figure 9). The entire grey area contained 4000 agents, 2000 of each type which are randomly placed at the start of the simulation. Agents within the model have the same preference, all want to be in an area where 50% or more of its neighbours are of the same type. Neighbourhood size was set to 200m to allow for agents on the fringes of geographical features to consider as neighbours agents on the other side of these fringes in the model where geographical features are not considered in neighbourhood calculations (i.e. buffers are not constrained by void area).

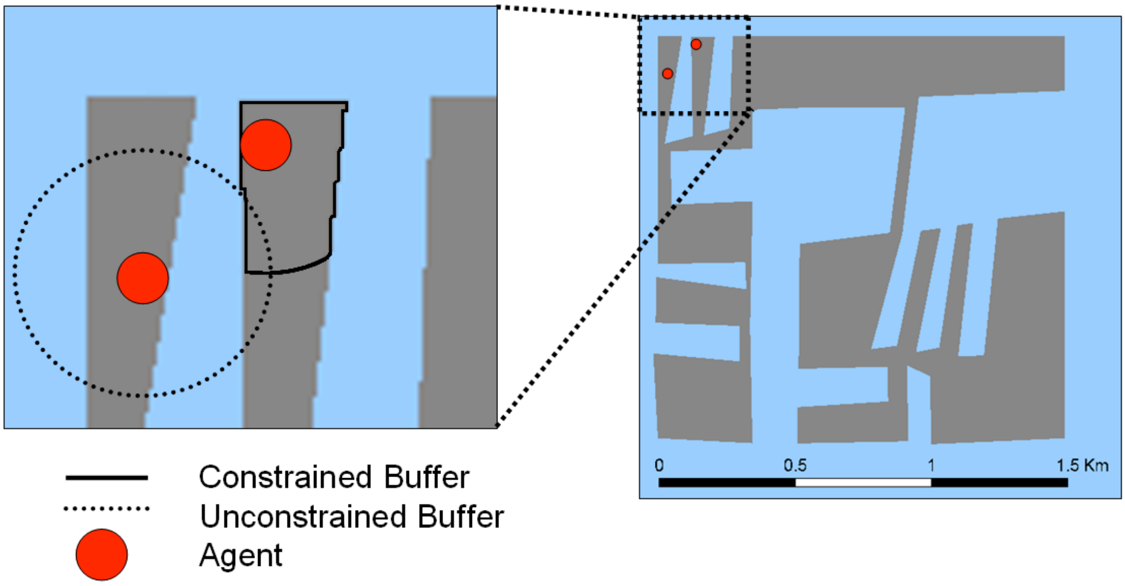


**Figure 9:** The area used to test how geographical features impact on the pattern of segregation and an example of constrained and unconstrained buffers used in neighbourhood calculations.

Figure 10 shows two final patterns of segregation, one from the basic segregation model and one from the model where geometry was not included. The patterns look quite similar suggesting the influence of geometry in neighbourhood calculations is quite subtle. The influence of geometry is most obvious at the fringes of geometrical boundaries, specifically the top left and centre of the images as shown in the two zoomed in areas of Figure 10. In the segregation model where geometry is not included in neighbourhood calculations and therefore the buffers are not constrained by the void space, one can clearly see agents are considering as neighbours, agents across the void on opposing spits and thus agents of different types are located in close proximity to each other while each others' preferences are met. Where geometry is considered, it can be seen that agents of different types are not located in the same areas as they would not consider agents on the opposite side of the void as neighbours and thus their preferences could potentially not be met which forces them to locate in more homogeneous neighbourhoods. The examination of the summary data presented in Table 4 highlights that the runs which included geometry stabilised more quickly than those without. However, the mean number of moves and the number times the agents moved are quite similar. From these simulations, one can see the effect that geometry has on neighbourhood formation which is not normally explicitly included in Schelling-type models.

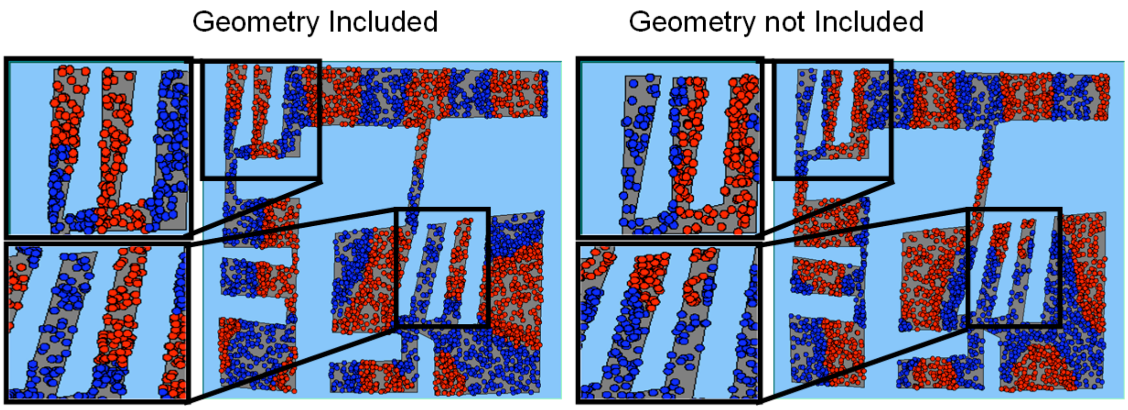


**Figure 10:** Screen captures of the final patterns of segregation when all agents are satisfied for different geometrical and non-geometrical segregation models.

**Table 4:** Comparison of geometrical and non-geometrical models: number of iterations and number of moves until all agents are satisfied.

| Model | Iterations until all Agents are Satisfied | | Moves | | | | | | | | | | | | | | Total Number of Agents that Moved | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 0 | | 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | | |
| | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev |
| No Geometry Included | 7.0 | 1.4 | 1940.8 | 36.2 | 1598.0 | 41.8 | 409.4 | 43.9 | 48.1 | 19.2 | 3.6 | 3.6 | 0.1 | 0.5 | 0.0 | 0.0 | 2059.2 | 36.2 |
| Geometry Included | 6.4 | 1.8 | 1925.8 | 65.8 | 1584.4 | 68.7 | 423.0 | 69.0 | 60.3 | 28.5 | 5.6 | 6.7 | 0.8 | 1.6 | 0.1 | 0.3 | 2074.2 | 65.8 |

## 4.4: Small Minority Populations

The simulations presented above have presumed 50/50 populations, which is rarely the case within cities. Often within populations, there are small minority groups which cluster in specific areas of the city; for example within London, the Bangladeshi population made up 2.1% (153900) of London's population (7172091) in 2001 (Greater London Authority, 2005) but of all the Bangladeshi population, 43% (65553) live within the Tower Hamlets municipality (Greater London Authority, 2004). The ability to model more than two groups thus allows one to explore differences between the numbers of dominant and subdominant groups within a population and we will extend to model this way here.

The following section explores a population of 4200, comprised of 37% red, 34% blue, 24% green and 5% white agents in 7 polygons as we show in Figure 11 where the white agents are only located in one area as highlighted. The background colour of the polygons represents the predominant social group of the area. For example, if the polygon is shaded red, there are more red agents in this area than any other type of agent, while a polygon shaded grey has equal numbers of at least two types of agents for example, red and blue. The first experiment was to see how this minority population group can change over a course of a simulation run. The experiment was carried out several times to explore how the pattern of segregation develops as the agents search for neighbourhoods where 50% or more of their neighbours are of the same types as themselves. Figure 12 highlights a typical simulation run with the majority of the white population staying in the same area when all agents are satisfied with the same areas they originated in. The number and type of predominant social groups in the area are on average the same as the model when first initialised as we indicate in Table 5.

| Polygon ID | Predominant social group | Total population | No. of red agents | No. of blue agents | No. of white agents | No. of green agents |
|---|---|---|---|---|---|---|
| 0 | Green | 600 | 180 | 180 | 0 | 240 |
| 1 | Red | 600 | 306 | 120 | 0 | 174 |
| 2 | Blue | 600 | 120 | 360 | 0 | 120 |
| 3 | Green | 600 | 60 | 30 | 210 | 300 |
| 4 | Blue | 600 | 180 | 360 | 0 | 60 |
| 5 | Red | 600 | 360 | 180 | 0 | 60 |
| 6 | Red | 600 | 360 | 180 | 0 | 60 |

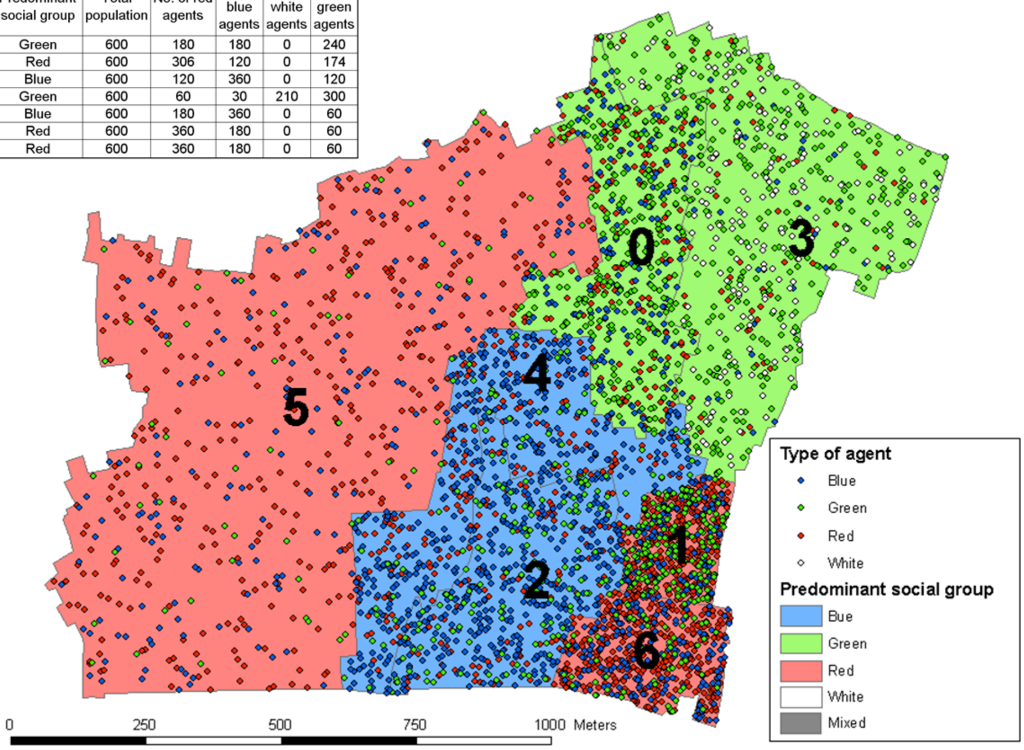


**Figure 11:** Initial starting conditions for the simulations where 5% of the population are white and agents are distributed randomly within their areas.

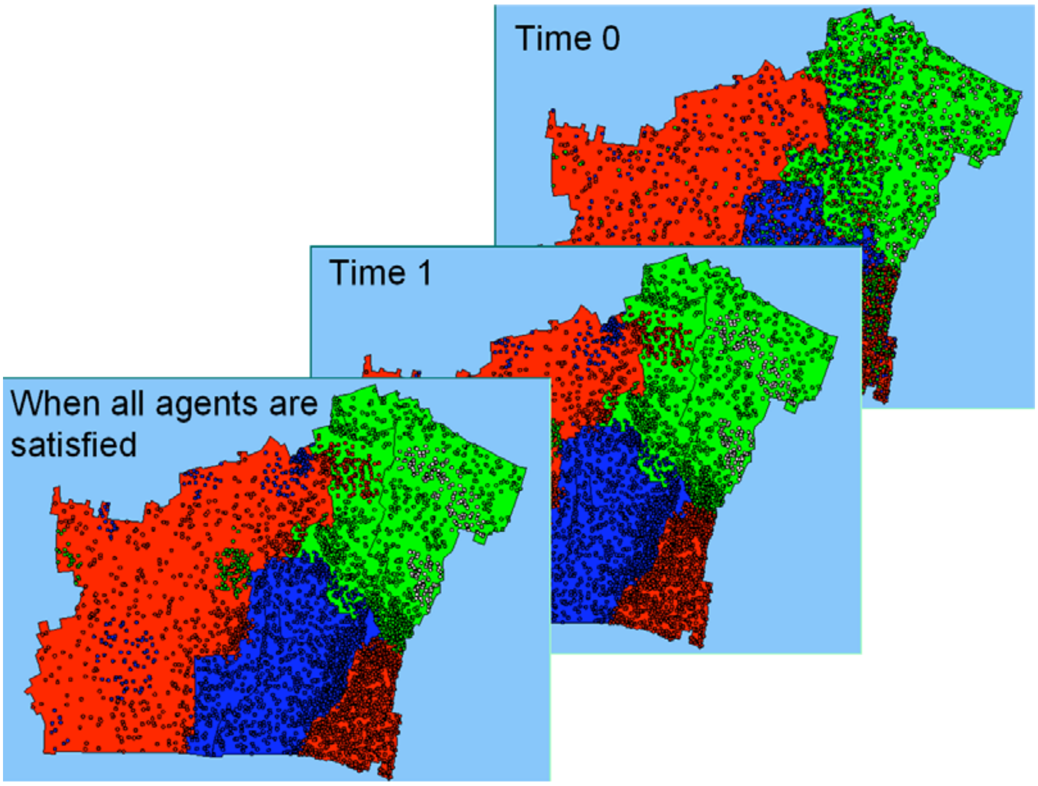


**Figure 12:** A typical simulation portraying the initial, first and last iteration until all agents are satisfied when only 5% of the population is white.

**Table 5:** Average iterations until all agents are satisfied when only 5% of the population is white.

| Initial no. of agents | | | | | | | | | Land-use at start | | | Iterations until all agents are satisfied | Moves | | | | Total no. of agents that moved | Land-use when all agents are satisfied | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Total | Red | | Blue | | Green | | White | | Red | Blue | Green | | 0 | 1 | 2 | 3 | | Red | Blue | Green |
| 4200 | 1566 | 37% | 1410 | 34% | 1014 | 24% | 210 | 5% | 3 | 2 | 2 | 5 | 1965 | 2122 | 110 | 3 | 2235 | 3 | 2 | 2 |

The next experiment was to explore how new agents affect this pattern of segregation. As with the previous experiment, the same initial conditions and preferences are used. However, within this experiment, 200 new agents were added at the end of each iteration and the model was run for 50 iterations (see Section 4.5 for a description of how the agents were added).

By the end of 50 iterations, the average land-use was red having the largest percentage in three areas where the blue and white agents make up the largest percentage in two areas each as we show in Table 6. It is noticeable that when the model was first initialised, there were two green areas. However, over the course of the simulation run as new agents are added, the green agents' dominance declines as white agents occupy the areas where the greens were once predominant. The white areas tended to be in the top right polygon where the initial white population was concentrated and in one of the adjacent polygons as its population grew. Figure 13 highlights a typical simulation run showing how clusters develop during the simulation where generally the addition of new agents reinforces these clusters. This is particularly the case where the white agents are clustered in one area. This area attracts new white agents forcing the area to grow and spill over to adjacent areas, reinforcing the predominant social group in that area and specifically forcing the green agents to lose their dominance in these areas.

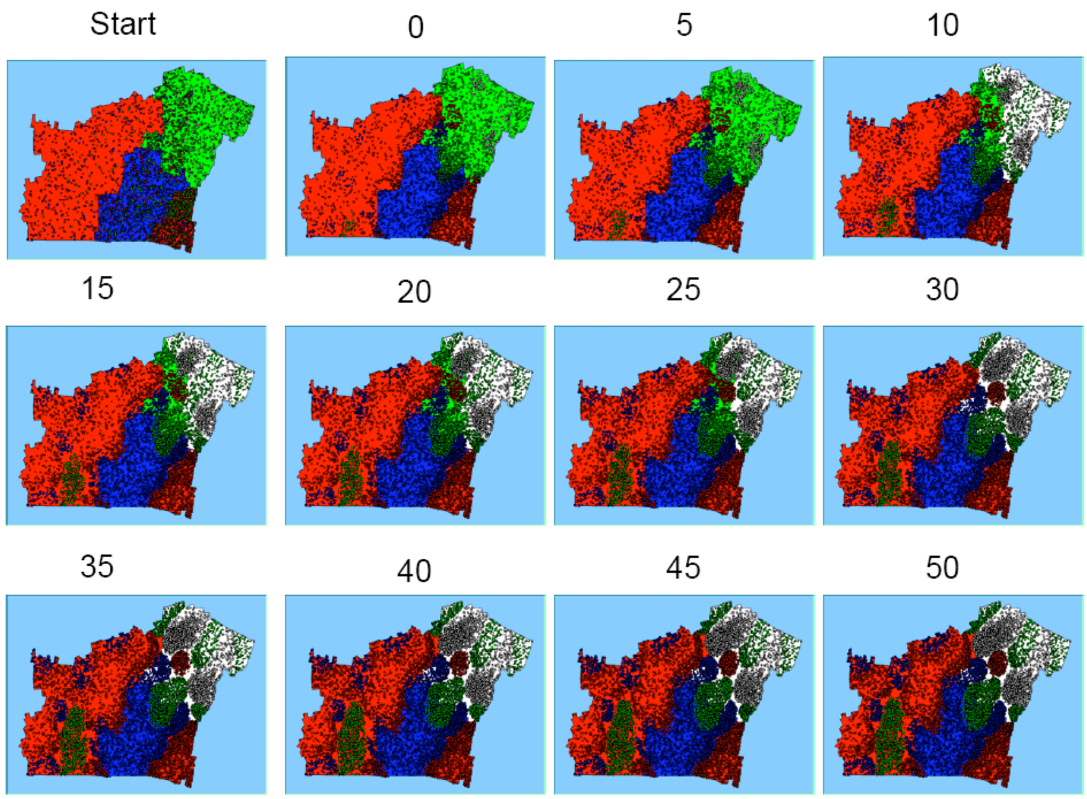


**Figure 13:** A typical simulation portrayed at every five time intervals where agents are being added when only 5% of the initial population is white.

**Table 6:** Average results when the initial population only contains 5% white and agents are added for 50 iterations.

| Initial no. of agents | | | | | | | | | Land-use at start | | | No. of agents at 50 iterations | | | | | | | | | Land-use at 50 iterations | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Total | Red | | Blue | | Green | | White | | Red | Blue | Green | Total | Red | | Blue | | Green | | White | | Red | Blue | Green | White |
| 4200 | 1566 | 37% | 1410 | 34% | 1014 | 24% | 210 | 5% | 3 | 2 | 2 | 14200 | 4053 | 29% | 3892 | 27% | 3512 | 25% | 2743 | 19% | 3 | 2 | 0 | 2 |

## 4.5: Addition and Removal of Agents

All cities and regions change both by growth and decline.  However, Fossett and Waren (2005) note that there is no mechanism for population turnover within Schelling's basic model, households are 'immortal' and thus a satisfied household can reside in the same location for ever.  The previous simulations were designed to study how established groups participate in constructing the city's social-spatial pattern. The following section highlights how an extra layer of dynamics can be added to the more traditional Schelling-type model, through the addition and removal of agents. The ability to add new agents and the death (removal) of older agents is not discussed in Schelling's original work for the topic is not heavily researched, as most Schelling-like models operate in cellular space where only a finite number of agents are possible depending on the size of the grid used.  However within the model developed here, agents are not restricted to one agent per cell.  This variation therefore allows one to explore how an area's social-spatial pattern might change through adding and removing agents, especially how new agents affect the composition of the area.  This could be considered as immigration and aging and the death of populations in urban areas.  It has similarities to the work of Portugali (2000) who simulated how agents could be added to the system and how neighbourhood patterns change due to the additions of agents.

The model itself varies from others presented thus far in a number of ways.  Firstly all the agents are given a new attribute: 'age'.  This age is determined randomly between 1 and 50 using a random number generator when the agent is first created at the beginning of the simulation.  At the end of each step (iteration) of the simulation, the agent's age increases by one.  In the previous version of the model, agents were only removed from the system if they could not find a suitable neighbourhood.  In this version, agents are also removed from the system when their age reaches 50.  The second variation to the model is the addition of new agents at the end of each step. These new agents are given a random social class and an age of 0 and placed randomly in the urban area when first created. The new agent then evaluates its neighbourhood and if dissatisfied, it moves to an area where its preferences are met. If the new agent cannot become satisfied with the area, it is removed from the system.

Within this test, agents are both added and removed from the system either because they are dissatisfied or die. The initial population was 700 agents (390 of type red (55.7%) and 310 of type blue (44.3%)). These agents were spread over several polygons as highlighted in Figure 14. Within the area, there were 3 areas where the predominant social group was red (i.e. 70% of the population was red), 1 area where the predominant social group was blue (70%) and 3 areas were classified as mixed (population composed of 50% of both groups). 100 new agents are added to the system at the end of each iteration and removed (when they reach the age of 50). As in previous models, each agent wanted to be in an area where 50% or more of its neighbours are of the same type. Neighbourhoods were defined as 100m radius around each particular agent.

| Polydon ID | Predominant social group | Total population | No. of red agents | No. of blue agents |
|---|---|---|---|---|
| 0 | Mixed | 100 | 50 | 50 |
| 1 | Red | 100 | 70 | 30 |
| 2 | Mixed | 100 | 50 | 50 |
| 3 | Red | 100 | 70 | 30 |
| 4 | Blue | 100 | 30 | 70 |
| 5 | Red | 100 | 70 | 30 |
| 6 | Mixed | 100 | 50 | 50 |

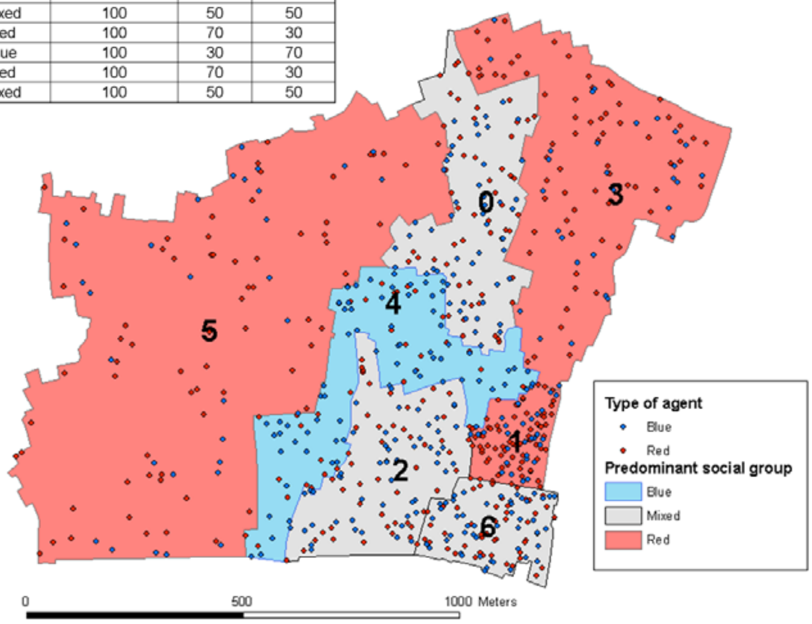


**Figure 14:** Initial starting conditions for the simulations with agents distributed randomly within their areas.

By 100 iterations, the percentage of both red and blue agents becomes approximately equal as shown in Table 7 and remains roughly constant. However the degree to which one group dominates one area varies over the course of each simulation as agents are added and removed due to death for in no instances are agents removed because they could not become satisfied with an area. Figure 15 traces a typical simulation, from the initial starting conditions and the pattern at every 50 iterations thereafter to the pattern at the end of the simulation is shown. As agents are added and removed from the system, the predominant social group of each area changes as can be seen from the underlying polygon colour in Figure 15. Red represents areas where population is 51% or more is red, blue areas where 51% or more of the population are blue, and grey areas represent equal numbers of red and blue agents.

Additionally Figure 15 highlights how clusters of groups do not stop at boundaries of areas. So while the aggregate data suggests that the area is predominantly of one type, clusters of distinct groups appear throughout the area and cross the boundaries between areas. These clusters would have been lost by purely using aggregate information. Even in the case where one area contains equal numbers of red and blue agents, distinct patterns of segregation are noticeable.

**Table 7:** Results from simulations where agents where both added and removed.

| Time | Number of Areas | | | | | | Number of Agents | | | | | | Number of Agents that have Died | | No. of Agents Satisfied with their Neighbourhood | | No. of Agents Dissatisfied with their Neighbourhood | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Red | | Blue | | Mixed | | Total | | Red | | Blue | | | | | | | |
| | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev | Mean | StDev |
| 50 | 3.5 | 0.9 | 3.4 | 0.9 | 0.1 | 0.3 | 2512.4 | 28.4 | 1254.1 | 27.3 | 1258.3 | 35.2 | 3187.6 | 28.4 | 2490.4 | 28.9 | 22.1 | 5.7 |
| 100 | 3.7 | 0.5 | 3.1 | 0.5 | 0.1 | 0.5 | 2486.4 | 28.8 | 1245.9 | 30.1 | 1240.6 | 28.3 | 5026.0 | 40.6 | 2466.1 | 28.9 | 20.3 | 4.9 |
| 150 | 3.7 | 1.1 | 3.2 | 1.1 | 0.1 | 0.3 | 2499.0 | 38.7 | 1255.7 | 31.4 | 1243.3 | 27.3 | 4987.4 | 47.6 | 2476.2 | 39.5 | 22.8 | 3.9 |
| 200 | 3.6 | 0.9 | 3.4 | 0.9 | 0.1 | 0.3 | 2496.5 | 26.6 | 1244.4 | 28.3 | 1252.1 | 34.7 | 5002.5 | 47.3 | 2473.0 | 26.0 | 23.5 | 6.5 |
| 250 | 3.9 | 0.8 | 3.1 | 0.8 | 0.0 | 0.0 | 2506.2 | 31.0 | 1259.0 | 26.1 | 1247.2 | 39.6 | 4990.3 | 40.7 | 2485.3 | 30.7 | 20.9 | 2.8 |
| 300 | 3.5 | 0.7 | 3.4 | 0.6 | 0.1 | 0.3 | 2496.4 | 27.8 | 1242.9 | 27.6 | 1253.6 | 23.7 | 5009.8 | 43.1 | 2472.6 | 32.1 | 23.8 | 5.5 |
| 350 | 3.3 | 1.2 | 3.7 | 1.2 | 0.0 | 0.0 | 2493.4 | 17.5 | 1245.1 | 26.1 | 1248.3 | 29.0 | 5003.0 | 26.7 | 2470.1 | 18.4 | 23.4 | 5.5 |
| 400 | 3.4 | 0.8 | 3.4 | 0.9 | 0.1 | 0.4 | 2501.8 | 33.3 | 1250.6 | 30.7 | 1251.2 | 28.3 | 4991.6 | 37.2 | 2479.4 | 34.0 | 22.4 | 4.5 |
| 450 | 3.6 | 1.1 | 3.2 | 1.0 | 0.1 | 0.4 | 2509.2 | 33.2 | 1251.9 | 24.0 | 1257.3 | 29.5 | 4992.6 | 45.7 | 2488.6 | 33.5 | 20.6 | 4.0 |
| 500 | 3.6 | 0.8 | 3.2 | 0.8 | 0.2 | 0.4 | 2499.0 | 22.0 | 1256.3 | 28.5 | 1242.7 | 22.7 | 5010.2 | 42.1 | 2475.9 | 21.9 | 23.1 | 5.5 |

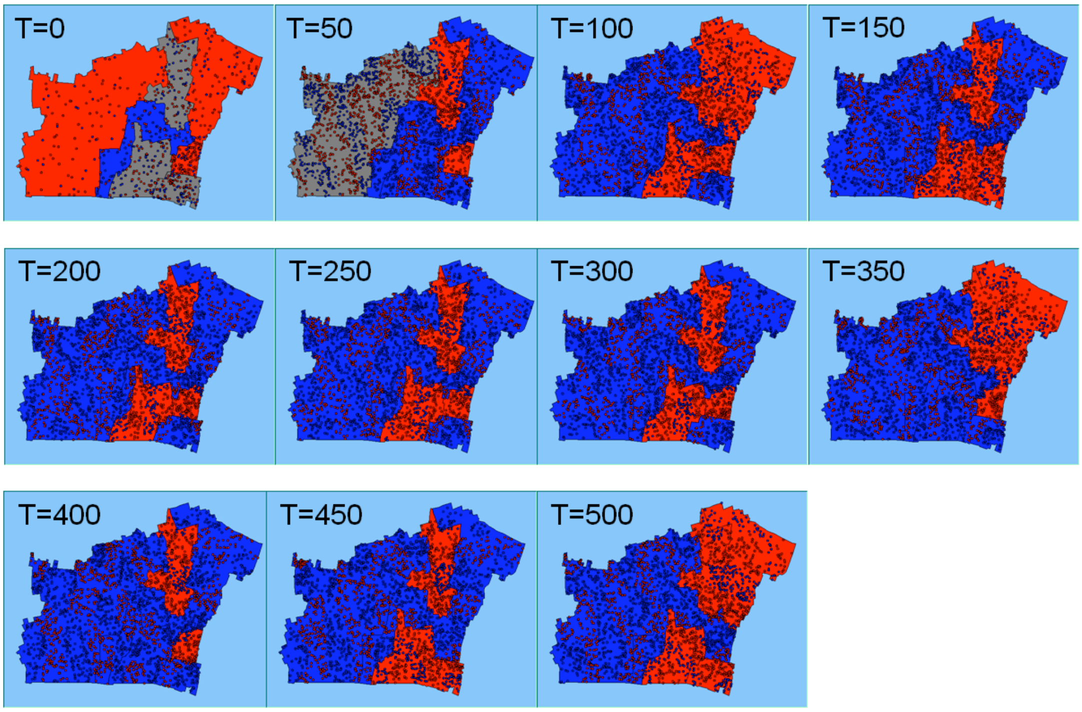

**Figure 15:** The effect of adding and removing agents to the system and the resulting patterns of segregation.

## 5: Discussion

The model presented in this paper is tightly bound to the vector GIS data model, and was developed due to certain limitations of representing space as a regular partition of squares akin to the traditional raster approach linking GIS and ABM. The model thus resolves specifically the lack of geometry, and the inability to represent objects of various shapes and sizes. We have also demonstrated that the environment and agents can be derived from GIS features by using the co-ordinate representation of each feature. Second, most relationships between agents can be evaluated within vector GIS using standard overlay operators such as point-in-polygon, buffering, intersection etc., making it possible to determine where agents are situated in relation to other agents and their environment. More specifically neighbourhood rules are available for evaluating adjacency, distance and so on. Third, GIS is an excellent tool for visualising and querying the outcomes of agent-based simulation. However, this is not to say that vector rather than raster representation is more appropriate for modelling. For example, Landis (2001) changed from vector-based polygons to raster-based grid cells in the representation used in the Californian Urban Futures

(CUF) models so that computation could be simplified. Additionally Benenson *et al.* (2005) comment that while vector GIS can represent urban objects in spatially explicit models, for theoretical models the points of a regular grid usually suffice. Therefore the purpose of the model needs to be considered.

The experiments not only highlight how individual actions can lead to more aggregate patterns emerging but how agents can be linked to geographic locations and how geometry can be incorporated directly into the simulation process. Furthermore the experiments allow for sensitivity testing of the model and to highlight the effect of the underlying model assumptions. This exploration provides a detailed understanding of the implications of each assumption but also allows one to evaluate the logic behind the model. This includes the influence of the size of neighbourhoods, the influence of geographical features and the degree to which segregation changes when agent preferences for neighbourhood composition change. These explorations showed that the geometry of an area can act as a physical barrier to segregation and that by increasing agents' preferences to reside in a specific group, marked segregation can emerge but not in a linear progression. A distinct shift in the degree of segregation occurs when agent preferences increase from 40% to 50% of their own type. As with the more 'traditional' segregation models, this model also highlights how with mild tastes and preferences to locate amongst 'like' demographic groups, segregation will emerge. Adding agents and removing agents from an existing population alters existing patterns but for new groups entering the system. The model illustrates how small minority groups cluster in areas and how these clusters remain persistent over time, outcomes which are well beyond what Schelling showed in his initial model.

The analysis of the results from the simulations in sections 4.4 and 4.5 demonstrate an important issue relating to the scale of analysis of segregation phenomena. In particular as we aggregate, we can unwittingly change the kinds of processes that agents enable. Aggregation can confuse our identification of coherent patterns that make sense in terms of basic human decision making. For example, in Figure 16A, we show a representative simulation outcome where all agents are satisfied with their current neighbourhood locations. While areas may have a predominant type of one agent within them (e.g. a polygon shaded red, say, has more red agents than any other type), there are areas where there are equal numbers of two or more groups (grey areas). However closer inspection of these mixed areas in Figure 16B reveals distinct micro clusters of different types of agents. Moreover it is also clear that clusters do not stop at boundaries but cross them as well and these clusters would be lost if we were only to consider aggregate level data without the ability of agents to move in free space.

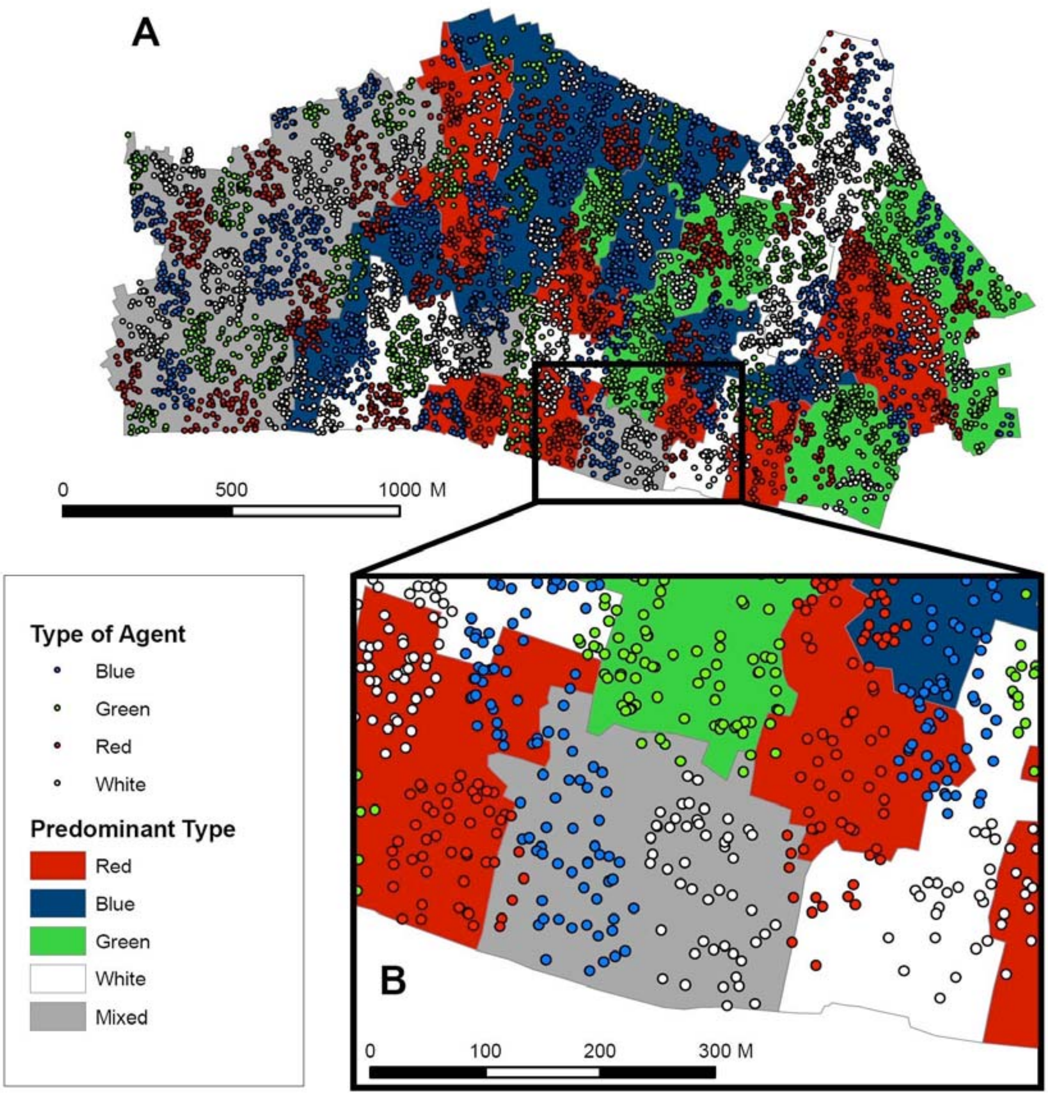


**Figure 16:** Segregation within areas and across boundaries.
A: The entire area, B: A Zoomed in section of A

Furthermore, this paper addresses one of the most important challenges facing ABM which is the need to share, communicate and disseminate not only model results but also to engender understanding of such models as well as their operation through an interactive environment for directly manipulating such models (Crooks *et al.*, 2007). In this, we are aided by advances in computation, particularly visualisation and networked communications. To reiterate the model, the source code and animations of the simulations are available at www.casa.ucl.ac.uk/abm/segregation. The models presented in this paper were purposely kept simple, mainly to explore how space and geometry impact on Schelling's segregation model and to highlight how this approach can be used to study urban phenomena. It is envisaged that the models can easily be extended from the source code by for example, including extra variables in the agent's choice of location such as economic preferences about an area.

Therefore the model can be considered as a representation and simulation of the urban environment as a series of points, lines and polygons. However, potentially it allows one to gain a greater understanding of small scale residential dynamics through the use of fine resolution data at the individual perspective, thus furthering our understanding of this complex issue. ABM is inherently suited to such a study as it allows the representation of an heterogeneous population with individual agents having different behaviours and characteristics. The ability to represent the world as a series of points, lines and polygons allows the inclusion of geometry into the modelling process, therefore allowing for different sizes of features such as houses, roads and so on to be portrayed. Linking people to place can be potentially achieved in the United Kingdom (UK) through the utilisation of the fine scale data sets, for example, using the national mapping agency's MasterMap TOIDs® to represent individual buildings, MasterMap's address layer to populate these building with a number of units, and assigning individual agents to each of these units. The agents ethnicity can be potentially extracted from the UK Electoral Role dataset (see Mateos *et al.*, 2007).

## 6: Conclusions

The model presented in this paper demonstrates how the representation of individuals through simple rules governing their behaviour and interaction at the macro-scale, can result in recognisable patterns at the macro-scale. The model highlights how theories and concepts pertaining to urban phenomena can easily be abstracted within geographically explicit agent-based models, helping further our understanding of how processes within cities operate. Furthermore, the model raises the importance of incorporating space and geometry when modelling urban systems. Additionally the approach the model takes allows us to relate closely to 'real' urban form while many other agent-based models use stylised forms to represent the urban environment.

The model departs from other typical models of segregation based on Schelling's ideas in a number of ways. To reiterate, most research in ABM pertaining to segregation has been carried out using the regular partition of space or using polygons to represent the location of households. However to date, little research has been carried out in relation to the importance of how the geometry of the environment affects the model outcomes. The model presented in this paper allows agents to move freely within the urban environment where their movement is not restricted to discrete empty cells or areas through the use of continuous representation of space – vector rather than raster space. The model focuses on the changing nature of segregation over space and time as agents move to new locations, and how segregated areas grow and decline over time. In this sense, it makes Schelling's model much more explicitly geographical than any other applications to date but it is easy to replicate and is an ideal basis for experimentation. The fact that it can be demonstrated using a whole range of media from pencil and paper to a variety of types of computation – on the desktop, the web etc., illustrates its pedagogic quality and the ease with which the model can be shared amongst non-experts as a demonstration of how complex, unexpected, and surprising patterns emerge from simple foundations.

We have also clearly outlined how the model has been implemented, making explicit the components of the model and the key mechanisms that drive the model findings. Clear description of how the model is implemented along with the source code helps with verification of the model, thus furthering our ability to model urban systems. The model provides the essential ingredients for cumulative scientific inquiry with a clearly specified model that facilitates replication and extension which is the key mission of traditional science.

## 7: References

Antonovics, K., Arcidiacono, P. and Walsh, R. (2003), *Games and Discrimination: Lessons From the Weakest Link*, Department of Economics, University of California at San Diego, Working Paper Series 2003-03, San Diego, CA.

Axelrod, R. (2007), Advancing the Art of Simulation in the Social Sciences, in Reynard, J.-P. (Ed.) *Handbook of Research on Nature Inspired Computing for Economy and Management*, Idea Group, Hershey, PA, pp 90-100.

Batty, M. (2005a), Approaches to Modelling in GIS: Spatial Representation and Temporal Dynamics, in Maguire, D.J., Batty, M. and Goodchild M, F. (eds.), *GIS, Spatial Analysis and Modelling*, ESRI Press, Redlands, CA, pp. 41-61

Batty, M. (2005b), *Cities and Complexity: Understanding Cities with Cellular Automata, Agent-Based Models, and Fractals,* The MIT Press, Cambridge, MA.

Batty, M., Barros, J. and Alves, S., Jr. (2004), *Cities: Continuity, Transformation and Emergence,* Centre for Advanced Spatial Analysis (University College London): Working Paper 72, London, UK.

Benenson, I. (1998), Multi-Agent Simulations of Residential Dynamics in a City, *Computers, Environment and Urban Systems,* 22(1): 25-42.

Benenson, I., Omer, I. and Hatna, E. (2002), Entity-Based Modelling of Urban Residential Dynamics: The Case of Yaffo, Tel Aviv, *Environment and Planning B: Planning and Design,* 29(4): 491-512.

Benenson, I. and Omer, I. (2003), High-Resolution Census Data: Simple Ways to Make Them Useful, *Data Science Journal (Spatial Data Usability Special Section),* 2: 117-127.

Benenson, I., Omer, I. and Hatna, E. (2003), Agent-Based Modelling of Householders' Migration Behaviour and Its Consequences, in Billari, F.C. and Prskawetz, A. (eds.), *Agent-Based Computational Demography: Using Simulation to Improve Our Understanding of Demographic Behaviour,* Physica-Verlag Heidelberg, New York, NY, pp. 97-115.

Benenson, I., Aronovich, S. and Noam, S. (2005), Let's Talk Objects: Generic Methodology for Urban High-Resolution Simulation, *Computers, Environment and Urban Systems,* 29(4): 425–453.

Benenson, I. and Torrens, P.M. (2004), *Geosimulation: Automata-Based Modelling of Urban Phenomena,* John Wiley & Sons, London, UK.

Bjornskau, T. (2005), Road Traffic Change: A Catalyst for Segregation? *Urban Studies,* 42(1): 69-89.

Brown, D. G. (2006), Agent-Based Models, in Gist, H. (ed.) *The Earths Changing Land: An Encyclopaedia of Land-Use and Land-Cover Change*, Greenwood Publishing Group, Westport, CN, pp. 7-13.

Bruch, E. (2006), Residential Mobility, Income Inequality, and Race/Ethnic Segregation in Los Angeles, *Population Association of America (PAA) 2006 Annual Meeting Program*, Los Angeles, CA, Available at http://paa2006.princeton.edu/download.aspx?submissionId=60143.

Bruch, E. E. and Mare, R. D. (2005), *Neighbourhood Choice and Neighbourhood Change*, California Centre for Population Research University of California – Los Angeles, Los Angeles, CA.

Castle, C. J .E. and Crooks, A. T. (2006), Principles and Concepts of Agent-Based Modelling for Developing Geospatial Simulations, Centre for Advanced Spatial Analysis (University College London), *Working Paper 110,* London.

Clark, W.A.V. (1991), Residential Preferences and Neighbourhood Racial Segregation: A Test of the Schelling Segregation Model, *Demography,* 28(1): 1-19.

Crooks, A. T. (2007), *Experimenting with Cities: Utilizing Agent-Based Models and GIS to Explore Urban Dynamics*, PhD Thesis, University College London.

Crooks, A. T., Castle, C. J .E. and Batty, M. (2007), Key Challenges in Agent-Based Modelling for Geo-Spatial Simulation, Centre for Advanced Spatial Analysis (University College London), *Working Paper 121*, London.

Conejeros, R. and Vargas, M. (2007), Group Effect, Productivity and Segregation Optimality, in Consiglio, A. (ed.) *Artificial Markets Modelling*, Springer Berlin Heidelberg, pp. 209-222.

Gilbert, N. and Terna, P. (2000), How to Build and Use Agent-Based Models in Social Science, *Mind & Society*, 1(1): 57-72.

Gimblett, H.R. (2002), *Integrating Geographic Information Systems and Agent-Based Modelling Techniques for Simulating Social and Ecological Processes,* Oxford University Press, Oxford, UK.

Gimblett, H.R., Richards, M.T. and Itami, R.M. (2002), Simulating Wildland Recreation Use and Conflicting Spatial Interactions using Rule-Driven Intelligent Agents, in Gimblett, H.R. (ed.) *Integrating Geographic Information Systems and Agent-Based Modelling Techniques for Simulating Social and Ecological Processes*, Oxford University Press, Oxford, UK, pp. 211-243.

Greater London Authority (GLA) (2004), *2001 Census Profiles: Bangladeshis in London,* Greater London Authority, London, UK.

Greater London Authority (GLA) (2005), *London - The World in a City: An Analysis of 2001 Census Results,* Greater London Authority, London, UK.

Flache, A. and Hegselmann, R. (2001), Do Irregular Grids Make a Difference? Relaxing the Spatial Regularity Assumption in Cellular Models of Social Dynamics, *Journal of Artificial Societies and Social Simulation,* 4(4).

Fossett, M. and Senft, R. (2004), SIMSEG and Generative Models: A Typology of Model-Generated Segregation Patterns, *Proceedings of the Agent 2004 Conference on Social Dynamics: Interaction, Reflexivity and Emergence,* Chicago, IL, pp. 39-78.

Fossett, M. and Waren, W. (2005), Overlooked Implications of Ethnic Preferences for Residential Segregation in Agent-Based Models, *Urban Studies,* 42(11): 1893-1917.

Helsley, R.W. and Strange, W.C. (1999), Gated Communities and the Economic Geography of Crime, *Journal of Urban Economics,* 46(1): 80-105.

JTS (2008), Java Topology Suite, Available at http://www.vividsolutions.com/jts/jtshome.htm [Accessed on 14th January, 2008].

Landis, J.D. (2001), CUF, CUFII, and CURBA: A Family of Spatially Explicit Urban Growth and Land-Use Policy Simulation Models, in Brail, R.K. and Klosterman, R.E. (eds.), *Planning Support Systems: Integrating Geographic Information Systems, Models and Visualisation Tools*, ESRI Press, Redlands, CA, pp. 157-200.

Laurie, A. J. and Jaggi, N. K. (2003), Role of 'Vision' in Neighbourhood Racial Segregation: A Variant of the Schelling Segregation Model, *Urban Studies*, 40(13): 2687–2704.

Massey, D.S. and Denton, N.A. (1993), *American Apartheid Segregation and the Making of the Underclass*, Harvard University Press, Cambridge, MA.

Mateos, P., Webber, R., and Longley, P.A. (2007), *The Cultural, Ethnic and Linguistic Classification of Populations and Neighbourhoods using Personal Names*, Centre for Advanced Spatial Analysis (University College London): Working Paper 116, London, UK.

Najlis, R. and North, M.J. (2004), Repast for GIS, in Macal, C. M., Sallach, D. and North, M.J. (eds.), *Proceedings of the Agent 2004 Conference on Social Dynamics: Interaction, Reflexivity and Emergence*, Chicago, IL, pp. 255-260.

Noonan, D.S. (2005), Neighbours, Barriers and Urban Environments: Are Things 'Different on the Other Side of the Tracks'? *Urban Studies,* 42(10): 1817-1835.

O'Sullivan, D. (2001), Exploring Spatial Process Dynamics using Irregular Cellular Automaton Models, *Geographical Analysis*, 33(1): 1-18.

O'Sullivan, D., MacGill, J. and Yu, C. (2003), Agent-Based Residential Segregation: A Hierarchically Structured Spatial Model, *Proceedings of Agent 2003 Conference on Challenges in Social Simulation*, The University of Chicago, Chicago, IL, pp. 493-507.

Omer, I. (2005), How Ethnicity Influences Residential Distributions: An Agent-Based Simulation, *Environment and Planning B: Planning and Design*, 32(5): 657-672.

OpenMap (2008), Open Systems Mapping Technology, Available at http://openmap.bbn.com/ [Accessed on 14th January, 2008].

Rabin, Y. (1987), The Roots of Segregation in the Eighties: The Role of Local Government Actions, in G.A. Tobin. (ed.) *Divided Neighbourhoods: Changing Patterns of Racial Segregation*, Sage Publications, London, UK, pp. 208–26.

Reardon, S.F. and O'Sullivan, D. (2004), Measures of Spatial Segregation, *Sociological Methodology*, 34: 121-162.

Repast (2008), Recursive Porous Agent Simulation Toolkit, Available at http://repast.sourceforge.net/repast_3/index.html [Accessed on 14th January, 2008].

Pancs, R. and Vriend, N.J. (2007), Schelling's spatial proximity model of segregation revisited, *Journal of Public Economics*, 91(1-2): 1-24.

Parker, D.C. (2005), Integration of Geographic Information Systems and Agent-Based Models of Land Use: Challenges and Prospects, in Maguire, D.J., Batty, M. and Goodchild M, F. (eds.), *GIS, Spatial Analysis and Modelling*, ESRI Press, Redlands, CA, pp. 403-422.

Portugali, J. (2000), *Self-Organization and the City*, Springer-Verlag, Berlin, Germany.

Schelling, T.C. (1969), Models of Segregation, *The American Economic Review*, 59: 488-493.

Schelling, T.C. (1971), Dynamic Models of Segregation, *Journal of Mathematical Sociology* 1: 143-186.

Schelling, T.C. (1978), *Micromotives and Macrobehavior*, WW Norton and Company, New York, NY.

Schelling, T.C. (2006), Some Fun, Thirty-Five Years Ago, in Tesfatsion, L. and Judd, K.L. (eds.), *Handbook of Computational Economics: Agent-Based Computational Economics*, North-Holland Publishing, Amsterdam, Netherlands, pp. 1639-1644.

Semboloni, F. (2000), The Growth of an Urban Cluster into a Dynamic, Self-Modifying Spatial Pattern, *Environment and Planning B*, 27(4): 549-564.

Shi, W. and Pang, M.Y.C. (2000), Development of Voronoi-Based Cellular Automata: An Integrated Dynamic Model for Geographical Information Systems, *International Journal of Geographical Information Science*, 14(5): 455-474.