Spacetime models in `R-INLA`

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Outline

Separable space-time models

PM-10 concentration in Piemonte, Italy

Kronecker product models

- $\mathbf{x} = \{x_{11}, ..., x_{n1}, x_{12}, ..., x_{nT}\}$
- assume

$$\pi(\mathbf{x}) \propto (|\mathbf{Q1} \otimes \mathbf{Q2}|^*)^{1/2} \exp\left(-\frac{1}{2}\mathbf{x}^T\{\mathbf{Q1} \otimes \mathbf{Q2}\}\mathbf{x}\right)$$

where $|.|^*$ is the generalized determinant

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- kronecker product model example in `R-INLA`

```
f(spatial, model='besagproper2',
  group=time, control.group=list(model='ar1'))
```

Spacetime interactions

- kronecker product models follows Clayton's rule
- combine **Q1** and **Q2** available
- warning care when main effects are in the model
- WARNING super care when **Q1** and/or **Q2** have rank deficiency
- the described dynamic model is type IV and uses **Q2** as AR(1)

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Space-time dynamic intercept

- The (linear) measurement equation

$$\mathbf{y}_{it} = F_{it}^{'}\beta + \mathbf{A}_{i(t)}\mathbf{x}_t + \epsilon_{it}$$

  - $\mathbf{F}_t$ is a matrix of covariates
  - $\boldsymbol{\beta}$ are the fixed effects
  - $\mathbf{A}_{(t)}$ picks out the appropriate values of $\mathbf{x}_t$
  - $\boldsymbol{\epsilon}_t \overset{\text{i.i.d.}}{\sim} N(0, \sigma^2\mathbf{I})$

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- vector AR(1) process for $\mathbf{x}$

$$\mathbf{x}_t = \rho\mathbf{x}_{t-1} + \boldsymbol{\omega}_t$$

  - $\boldsymbol{\omega}_{\mathbf{t}}$: spatial SPDE model

$$\boldsymbol{\omega}_t \overset{\text{i.i.d.}}{\sim} N(\mathbf{0}, \mathbf{Q}^{-1}),$$

  - ρ is the time correlation

PM-10 concentration in Piemonte, Italy

Cameletti *et al.* (2011), on `r-inla.org`

- 24 monitoring stations
- Daily data from 10/05 to 03/06

Space model part

- Make the mesh

```
mesh <- inla.mesh.2d(points =NULL,
                     points.domain=borders,
                     offset=c(10, 140),
                     max.edge=c(40,1000))
```

- Make the latent model

```
spde = inla.create.spde(mesh,model="matern")
```

Using the group feature

- Construct a kronecker product model using the group feature

```
formula = y ~ -1 + intercept + WS + HMIX + ... +
   f(field, model=spde,
      group =time,
      control.group=list(model="ar1")
     )
```

- This tells INLA that the observations are grouped in a certain way.
- `control.group` contains the grouping model (`ar1`, `exchangable`, `rw1`, and others) as well as their prior specifications.

Make an **A** matrix

- Use the `group` argument

```
LocationMatrix = inla.spde.make.A(mesh = mesh,
          loc =dataLoc, group=time, n.group=nT)
```

- data locations in all `group=time` level
- builds an **A** matrix in an appropriate way

Organising the data

Covariates at the data points, but the latent field only defined their through the A matrix

We need to make sure that **A** *only applies to the random effect.*

```
idx.set <- inla.spde.make.index("mesh.idx",n.field=nmesh,
                  n.group=T)
stack = inla.stack( data = dat,
       A = list(1, LocationMatrix),
       effects = list( list(WS = cov$WS,...),
           c(idx.set,
                        list(intercept=rep(1,mesh$n*nT)))
       )
    )
```