12 ♥ 11 ♥ 15

Lista 2

1) $X_1, \dots, X_n$ a.a. $N(0, \sigma^2)$

$H_0: \sigma^2 = \sigma_0^2$ vs $H_1: \sigma^2 > \sigma_0^2$

$$L(\theta, \underset{\sim}{x}) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\sum x_i^2/2\sigma^2}$$

$H_0: \sigma^2 = \sigma_0^2$ $\quad H_1: \sigma^2 = \sigma_1^2$ $\quad$ (com $\sigma_1^2 > \sigma_0^2$)

$$\frac{L_1(\theta, \underset{\sim}{x})}{L_0(\theta, \underset{\sim}{x})} = \frac{\left(1/\sigma_1\sqrt{2\pi}\right)^n e^{-\sum x_i^2/2\sigma_1^2}}{\left(1/\sigma_0\sqrt{2\pi}\right)^n e^{-\sum x_i^2/2\sigma_0^2}} = \left(\frac{\sigma_0}{\sigma_1}\right)^n e^{-\frac{\sum x_i^2}{2}\left[1/\sigma_1^2 - 1/\sigma_0^2\right]}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \left(\frac{\sigma_0}{\sigma_1}\right)^n e^{-1/2 \sum x_i^2 \left[1/\sigma_1^2 - 1/\sigma_0^2\right]} \geq K \right\}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; e^{-1/2 \sum x_i^2 \left[1/\sigma_1^2 - 1/\sigma_0^2\right]} \geq K_1 \right\}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; -\frac{1}{2} \sum x_i^2 \overbrace{\left[\frac{1}{\sigma_1^2} - \frac{1}{\sigma_0^2}\right]}^{<0} \geq K_2 \right\}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \sum x_i^2 \geq c \right\}$$

2) $X_1, \dots, X_n$ a.a. $X \sim \text{Bernoulli}(\theta)$

$H_0: \theta = \theta_0$ vs $H_1: \theta = \theta_1$ $\quad (\theta_1 > \theta_0)$

$$f(x|\theta) = \theta^x (1-\theta)^{1-x} \qquad 0 < \theta < 1 \qquad x = 0,1$$

$$L(\theta, \underset{\sim}{x}) = \theta^{\sum x_i} (1-\theta)^{n - \sum x_i}$$

$$\frac{L_1(\underset{\sim}{x})}{L_0(\underset{\sim}{x})} = \frac{\theta_1^{\sum x_i} (1-\theta_1)^{n-\sum x_i}}{\theta_0^{\sum x_i} (1-\theta_0)^{n-\sum x_i}} = \left(\frac{\theta_1}{\theta_0}\right)^{\sum x_i} \left(\frac{1-\theta_1}{1-\theta_0}\right)^n \left(\frac{1-\theta_0}{1-\theta_1}\right)^{\sum x_i} \geq K$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \left[\frac{\theta_1}{\theta_0}\left(\frac{1-\theta_0}{1-\theta_1}\right)\right]^{\sum x_i} \geqslant K_1 \right\}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \sum x_i \log\left[\frac{\theta_1}{\theta_0}\left(\frac{1-\theta_0}{1-\theta_1}\right)\right] \geqslant \log K_1 \right\}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \sum x_i \geqslant c \right\}$$

3) $f(x|\theta) = \theta^2 x e^{-\theta x}$ $\quad x > 0 \quad \theta > 0$

$H_0: \theta = 1$ vs $H_1: \theta = 2$ $\qquad f(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ $\quad \Gamma(\alpha,\beta)$

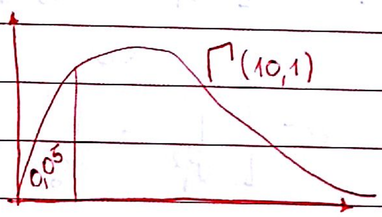
RC $n = 5$ e $\alpha = 0{,}05$ $\qquad 2 \quad \theta$

$L(\theta, \underset{\sim}{x}) = \theta^{2n} \left(\prod_{i=1}^{n} x_i\right) e^{-\theta \sum x_i}$ $\qquad X \sim \Gamma(2, \theta)$

$$\frac{L_1(\underset{\sim}{x})}{L_0(\underset{\sim}{x})} = \frac{\overbrace{1024}^{2^{10}} (\prod x_i)\, e^{-2\sum x_i}}{(\prod x_i)\, e^{-\sum x_i}} = 1024\, e^{-\sum x_i}$$

$$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; 1024\, e^{-\sum x_i} \geqslant K \right\} = \left\{ \underset{\sim}{x} \;\middle|\; -\sum x_i \geqslant c \right\} = \left\{ \underset{\sim}{x} \;\middle|\; \sum x_i \leqslant c' \right\}$$

$\sum_{i=1}^{5} X_i \sim \Gamma(10, \theta)$



$\alpha = 0{,}05 = P\left(\sum_{i=1}^{5} X_i \leqslant c \;\middle|\; \theta = 1\right)$

$c = 5{,}425 \Rightarrow A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \sum x_i \leqslant 5{,}425 \right\}$

4) $X_1, \ldots, X_n$ a.a. Poisson (θ) $\qquad f(x|\theta) = \frac{e^{-\theta}\theta^x}{x!}$

Teste UMP para $H_0: \theta = \theta_0$ vs $H_1: \theta > \theta_0$

$L(\theta, \underset{\sim}{x}) = \frac{\theta^{\sum x_i} e^{-n\theta}}{\prod x_i!}$

$$\frac{L_1(\theta, \underset{\sim}{x})}{L_0(\theta, \underset{\sim}{x})} = \frac{\dfrac{\theta_1^{\sum x_i} e^{-n\theta_1}}{\cancel{\prod x_i!}}}{\dfrac{\theta_0^{\sum x_i} e^{-n\theta_0}}{\cancel{\prod x_i!}}} = \left(\frac{\theta_1}{\theta_0}\right)^{\sum x_i} e^{-n(\theta_1 - \theta_0)}$$

$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \left(\frac{\theta_1}{\theta_0}\right)^{\sum x_i} e^{-n(\theta_1-\theta_0)} \geq K \right\} = \left\{ \underset{\sim}{x} \;\middle|\; \left(\frac{\theta_1}{\theta_0}\right)^{\sum x_i} \geq K_1 \right\}$

$A_1^* = \left\{ \underset{\sim}{x} \;\middle|\; \sum x_i \underbrace{\log\left(\frac{\theta_1}{\theta_0}\right)}_{>0} \geq K_2 \right\} = \left\{ \underset{\sim}{x} \;\middle|\; \sum X_i \geq c \right\}$

A_1^* é MP para $H_0: \theta = \theta_0$ $H_1: \theta = \theta_1$

A_1^* é UMP para $H_0: \theta = \theta_0$ $H_1: \theta > \theta_1$