Contents

1 Probability Theory 1
  1.1 Set Theory 1
  1.2 Basics of Probability Theory 5
    1.2.1 Axiomatic Foundations 5
    1.2.2 The Calculus of Probabilities 9
    1.2.3 Counting 13
    1.2.4 Enumerating Outcomes 16
  1.3 Conditional Probability and Independence 20
  1.4 Random Variables 27
  1.5 Distribution Functions 29
  1.6 Density and Mass Functions 34
  1.7 Exercises 37
  1.8 Miscellanea 44

2 Transformations and Expectations 47
  2.1 Distributions of Functions of a Random Variable 47
  2.2 Expected Values 55
  2.3 Moments and Moment Generating Functions 59
  2.4 Differentiating Under an Integral Sign 68
  2.5 Exercises 76
  2.6 Miscellanea 82

3 Common Families of Distributions 85
  3.1 Introduction 85
  3.2 Discrete Distributions 85
  3.3 Continuous Distributions 98
  3.4 Exponential Families 111
  3.5 Location and Scale Families 116
CONTENTS

3.6 Inequalities and Identities 121
  3.6.1 Probability Inequalities 122
  3.6.2 Identities 123
3.7 Exercises 127
3.8 Miscellanea 135

4 Multiple Random Variables 139
  4.1 Joint and Marginal Distributions 139
  4.2 Conditional Distributions and Independence 147
  4.3 Bivariate Transformations 156
  4.4 Hierarchical Models and Mixture Distributions 162
  4.5 Covariance and Correlation 169
  4.6 Multivariate Distributions 177
  4.7 Inequalities
    4.7.1 Numerical Inequalities 186
    4.7.2 Functional Inequalities 189
  4.8 Exercises 192
  4.9 Miscellanea 203

5 Properties of a Random Sample 207
  5.1 Basic Concepts of Random Samples 207
  5.2 Sums of Random Variables from a Random Sample 211
  5.3 Sampling from the Normal Distribution
    5.3.1 Properties of the Sample Mean and Variance 218
    5.3.2 The Derived Distributions: Student’s t and Snedecor’s F 222
  5.4 Order Statistics 226
  5.5 Convergence Concepts
    5.5.1 Convergence in Probability 232
    5.5.2 Almost Sure Convergence 234
    5.5.3 Convergence in Distribution 235
    5.5.4 The Delta Method 240
  5.6 Generating a Random Sample
    5.6.1 Direct Methods 247
    5.6.2 Indirect Methods 251
    5.6.3 The Accept/Reject Algorithm 253
  5.7 Exercises 255
  5.8 Miscellanea 267

6 Principles of Data Reduction 271
  6.1 Introduction 271
  6.2 The Sufficiency Principle
    6.2.1 Sufficient Statistics 272
    6.2.2 Minimal Sufficient Statistics 279
    6.2.3 Ancillary Statistics 282
    6.2.4 Sufficient, Ancillary, and Complete Statistics 284
## CONTENTS

6.3 The Likelihood Principle \[ \text{290} \]
- 6.3.1 The Likelihood Function \[ \text{290} \]
- 6.3.2 The Formal Likelihood Principle \[ \text{292} \]
6.4 The Equivariance Principle \[ \text{296} \]
6.5 Exercises \[ \text{300} \]
6.6 Miscellanea \[ \text{307} \]

7 **Point Estimation** \[ \text{311} \]
- 7.1 Introduction \[ \text{311} \]
- 7.2 Methods of Finding Estimators \[ \text{312} \]
  - 7.2.1 Method of Moments \[ \text{312} \]
  - 7.2.2 Maximum Likelihood Estimators \[ \text{315} \]
  - 7.2.3 Bayes Estimators \[ \text{324} \]
  - 7.2.4 The EM Algorithm \[ \text{326} \]
- 7.3 Methods of Evaluating Estimators \[ \text{330} \]
  - 7.3.1 Mean Squared Error \[ \text{330} \]
  - 7.3.2 Best Unbiased Estimators \[ \text{334} \]
  - 7.3.3 Sufficiency and Unbiasedness \[ \text{342} \]
  - 7.3.4 Loss Function Optimality \[ \text{348} \]
- 7.4 Exercises \[ \text{355} \]
- 7.5 Miscellanea \[ \text{367} \]

8 **Hypothesis Testing** \[ \text{373} \]
- 8.1 Introduction \[ \text{373} \]
- 8.2 Methods of Finding Tests \[ \text{374} \]
  - 8.2.1 Likelihood Ratio Tests \[ \text{374} \]
  - 8.2.2 Bayesian Tests \[ \text{379} \]
  - 8.2.3 Union–Intersection and Intersection–Union Tests \[ \text{380} \]
- 8.3 Methods of Evaluating Tests \[ \text{382} \]
  - 8.3.1 Error Probabilities and the Power Function \[ \text{382} \]
  - 8.3.2 Most Powerful Tests \[ \text{387} \]
  - 8.3.3 Sizes of Union–Intersection and Intersection–Union Tests \[ \text{394} \]
  - 8.3.4 p-Values \[ \text{397} \]
  - 8.3.5 Loss Function Optimality \[ \text{400} \]
- 8.4 Exercises \[ \text{402} \]
- 8.5 Miscellanea \[ \text{413} \]

9 **Interval Estimation** \[ \text{417} \]
- 9.1 Introduction \[ \text{417} \]
- 9.2 Methods of Finding Interval Estimators \[ \text{420} \]
  - 9.2.1 Inverting a Test Statistic \[ \text{420} \]
  - 9.2.2 Pivotal Quantities \[ \text{427} \]
  - 9.2.3 Pivoting the CDF \[ \text{430} \]
  - 9.2.4 Bayesian Intervals \[ \text{435} \]
9.3 Methods of Evaluating Interval Estimators 440
  9.3.1 Size and Coverage Probability 440
  9.3.2 Test-Related Optimality 444
  9.3.3 Bayesian Optimality 447
  9.3.4 Loss Function Optimality 449

9.4 Exercises 451

9.5 Miscellanea 463

10 Asymptotic Evaluations 467
  10.1 Point Estimation 467
    10.1.1 Consistency 467
    10.1.2 Efficiency 470
    10.1.3 Calculations and Comparisons 473
    10.1.4 Bootstrap Standard Errors 478
  10.2 Robustness 481
    10.2.1 The Mean and the Median 482
    10.2.2 M-Estimators 484
  10.3 Hypothesis Testing 488
    10.3.1 Asymptotic Distribution of LRTs 488
    10.3.2 Other Large-Sample Tests 492
  10.4 Interval Estimation 496
    10.4.1 Approximate Maximum Likelihood Intervals 496
    10.4.2 Other Large-Sample Intervals 499
  10.5 Exercises 504
  10.6 Miscellanea 515

11 Analysis of Variance and Regression 521
  11.1 Introduction 521
  11.2 Oneway Analysis of Variance 522
    11.2.1 Model and Distribution Assumptions 524
    11.2.2 The Classic ANOVA Hypothesis 525
    11.2.3 Inferences Regarding Linear Combinations of Means 527
    11.2.4 The ANOVA F Test 530
    11.2.5 Simultaneous Estimation of Contrasts 534
    11.2.6 Partitioning Sums of Squares 536
  11.3 Simple Linear Regression 539
    11.3.1 Least Squares: A Mathematical Solution 542
    11.3.2 Best Linear Unbiased Estimators: A Statistical Solution 544
    11.3.3 Models and Distribution Assumptions 548
    11.3.4 Estimation and Testing with Normal Errors 550
    11.3.5 Estimation and Prediction at a Specified $x = x_0$ 557
    11.3.6 Simultaneous Estimation and Confidence Bands 559
  11.4 Exercises 563
  11.5 Miscellanea 572
## CONTENTS

**12 Regression Models**  577  
12.1 Introduction  577  
12.2 Regression with Errors in Variables  577  
   12.2.1 Functional and Structural Relationships  579  
   12.2.2 A Least Squares Solution  581  
   12.2.3 Maximum Likelihood Estimation  583  
   12.2.4 Confidence Sets  588  
12.3 Logistic Regression  591  
   12.3.1 The Model  591  
   12.3.2 Estimation  593  
12.4 Robust Regression  597  
12.5 Exercises  602  
12.6 Miscellanea  608  

Appendix: Computer Algebra  613  

Table of Common Distributions  621  

References  629  

Author Index  645  

Subject Index  649