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MEASURING AND TESTING THE IMPACT OF NEWS ON VOLATILITY

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ABSTRACT

This paper introduces the News Impact Curve to measure how new information is incorporated into volatility estimates. A variety of new and existing ARCH models are compared and estimated with daily Japanese stock return data to determine the shape of the News Impact Curve. New diagnostic tests are presented which emphasize the asymmetry of the volatility response to news. A partially non-parametric ARCH model is introduced to allow the data to estimate this shape. A comparison of this model with the existing models suggests that the best models are one by Glosten Jaganathan and Runkle (GJR) and Nelson's EGARCH. Similar results hold on a pre--crash sample period but are less strong.

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MEASURING AND TESTING THE IMPACT OF NEWS ON VOLATILITY

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It is now well established that volatility is predictable in most financial markets. In a recent survey by Bollerslev et al (1990) over 200 articles were cited which estimated or examined ARCH or alternative models of time varying heteroskedasticity. With this growth in interest and applications, there has also grown a literature on alternative models which are designed to allow different features of the data to be reflected in the model. Some of these are tightly parameteric models while others are non-parametric in spirit.

In this paper, we suggest a new metric with which these volatility models can be compared. We discuss some of the alternative models which are being tried and introduce several models of our own which should nest many of the existing models. We will also suggest several new diagnostic tests for volatility models.

In the next section, we discuss several models of predictable volatility and introduce the idea of a News Impact Curve which characterizes the impact of innovations on volatility implicit in a volatility model. In section III, we suggest several new diagnostic tests based on the News Impact Curve. In section IV, a partially non-parametric ARCH model is introduced. Section V presents and compares empirical estimates of several volatility models using a Japanese stock returns series. The new diagnostic tests are employed to check the adequacy of the models. In section VI the partially non-parametric model is estimated and compared with the others, and in section VII, the best models are reestimated on a pre-crash sample period. Section VIII concludes the paper.

II: MODELS OF PREDICTABLE VOLATILITY

The conditional mean and variance of a time series $\{y_t\}$ given a past information set, \mathscr{F}_{t-1} , are defined by

(1)
$$\mathbf{m}_t = \mathbf{E}(\mathbf{y}_t | \mathscr{T}_{t-1}), \quad \mathbf{h}_t = \mathbf{V}(\mathbf{y}_t | \mathscr{T}_{t-1}), \quad \boldsymbol{\epsilon}_t = \mathbf{y}_t - \mathbf{m}_t$$

where h_t is in general a non-negative random variable. The precise parametrization of this conditional variance function is however a matter of econometric specification just as is the specification of the mean.

In Engle (1982) several alternative formulations were discussed but the one developed in most detail was the p^{th} order autoregressive model:

(2)
$$h_t = \omega + \sum_{i=1}^p \alpha_i \epsilon_{t-i}^2$$

which was generalized to the GARCH(p,q) model by Bollerslev (1986):

(3)
$$\mathbf{h}_{t} = \omega + \sum_{i=1}^{p} \alpha_{i} \epsilon_{t-i}^{2} + \sum_{i=1}^{q} \beta_{i} \mathbf{h}_{t-i}$$

Apparently, the GARCH model is an infinite order ARCH model and often provides a highly parsimonious lag shape. Empirically these models have been very successful with the GARCH(1,1) the general favorite in the vast majority of cases. Furthermore, these applications typically reveal that there is a long term persistence in the effects of shocks in period t onto the conditional volatility in period t+s for large s. That is, there typically appears to be a unit root in the autoregressive polynomial associated with (2) or (3).

In spite of the apparent success of these simple parametrizations, there are some features of the data which these models are unable to pick out. The most interesting of these is the "leverage" effect emphasized by Nelson (1990) based on an argument by Black(1976). Statistically, this effect says that negative surprises to asset markets increase predictable volatility more than positive surprises. Thus the conditional variance function ought not be constrainted to be symmetric in past ϵ 's. Nelson (1990) proposed the exponential GARCH or EGARCH model:

(4)
$$\log h_{t} = \omega + \beta \cdot \log h_{t-1} + \gamma \cdot \frac{\epsilon_{t-1}}{h_{t-1}^{1/2}} + \alpha \left[\frac{|\epsilon_{t-1}|}{h_{t-1}^{1/2}} - (2/\pi)^{1/2} \right]$$

which is asymmetric because the level of ϵ relative to its standard deviation is included with coefficient γ which would typically be negative reflecting that positive shocks generate less volatility all else being equal. Shocks are measured relative to their standard deviations. The use of absolute errors and logs increases the impact of large shocks on the next period conditional variance.

A comparison between the GARCH(1,1) model and the EGARCH(1,1) suggests an interesting metric in which to analyze all general forms of conditional heteroskedasticity. How does new information affect the next period variance? Holding constant the information dated t-2 and earlier, one can examine the implied relation between ϵ_{t-1} and h_t . This curve might be called the <u>News Impact Curve</u>. In the GARCH model this curve is a quadratic centered on $\epsilon_{t-1}=0$. For the EGARCH, it has its minimum at $\epsilon_{t-1}=0$ and is exponentially increasing in both directions but with different parameters. In particular, the relation for the EGARCH evaluated at a conditional variance h is:

(5)
$$h_{t} = A \cdot \exp\left[\frac{(\alpha + \gamma)}{h^{1/2}} \cdot \epsilon_{t-1}\right], \text{ for } \epsilon_{t-1} > 0, \text{ and}$$
$$h_{t} = A \cdot \exp\left[\frac{(\gamma - \alpha)}{h^{1/2}} \cdot \epsilon_{t-1}\right], \text{ for } \epsilon_{t-1} < 0$$
where, $A \equiv h \cdot \exp[\omega - \alpha(2/\pi)^{1/2}]$

In figure 1, these are compared with the GARCH(1,1) for $\gamma < 0$ but $\alpha + \gamma > 0$. If the curves were extrapolated, the EGARCH would have the higher variance in both directions as the exponential curve eventually dominates the quadratic. Thus, from the News Impact Curve, the EGARCH model differs from the standard GARCH model in two main respects: [1] EGARCH allows good news and bad news to have a different impact on volatility while the standard GARCH model does not, and [2] EGARCH allows big news to have much more impact on volatility than the standard GARCH model.



FIGURE 1

The News Impact Curve can be examined for many other models. A great many of the alternatives are symmetric such as Schwert's (1990) standard deviation model:

(6)
$$\mathbf{h}_{\mathbf{t}} = \left[\omega + \sum_{i=1}^{p} \alpha_{i} |\epsilon_{\mathbf{t}-i}|\right]^{2}$$

which is quadratic in the news as is Bera and Lee (1989) augmented ARCH model. Other symmetric functions are implied by Engle and Bollerslev (1986) who fit both

(7)
$$h_{t} = \omega + \alpha |\epsilon_{t-1}|^{\gamma} + \beta h_{t-1}$$

(8)
$$h_{t} = \omega + \alpha [2\phi(\epsilon_{t-1}/\delta)-1] + \beta h_{t-1}$$
where $\phi(\cdot)$ is the cumulative density function of a standard normal

If $\gamma < 2$ and if $\delta > 0$, then both have a News Impact Curve which is symmetric but with reduced response to extreme news. This was found empirically. The multiplicative ARCH model of Mihoj (1987) and Geweke (1986) and Pantula (1986) relates the log of h to the log of lagged squared residuals. Thus the news impact response curve is given by:

(9)
$$h_t = A \cdot \epsilon_{t-1}^{2\alpha}$$
 where A is a constant

which therefore has the same shape as that in (7) although it goes automatically through the origin which appears to be a drawback to this model discussed by Engle and Bollerslev (1986a). Similarly, Higgins and Bera (1990) introduce a non-linear ARCH model which has a constant elasticity of substitution between terms in an ordinary ARCH model or GARCH model. Again this implies a power function as in (7) and (9). Finally, Friedman and Kuttner (1990) introduce a modified ARCH model which gives smaller coefficients to large residuals for the purpose showing that large shocks have lower persistence than small shocks. A similar finding was illustrated by the options pricing results in Engle and Mustafa (1989) and Schwert (1990). In each case, the extreme portions of the News Impact Curve are reduced in this specification.

To allow an asymmetric impact response curve centered at a non-zero ϵ_{t-1} , other extensions are needed. Possibly the simplest is proposed by Engle (1990) which simply allows the minimum of the News Impact Curve for the GARCH model to lie other than at the origin. The model called the asymmetric GARCH or AGARCH model is

(10)
$$h_{t} = \omega + \alpha (\epsilon_{t-1} + \gamma)^{2} + \beta h_{t-1}$$
$$= \omega^{*} + \alpha \epsilon_{t-1}^{2} + \gamma^{*} \epsilon_{t-1} + \beta h_{t-1}$$

where $\omega^* = \omega + \alpha \gamma^2$ and $\gamma^* = 2\alpha \gamma$. By the Black or leverage effect we expect and find the γ and γ^* are negative so that the minimum of the News Impact Curve lies to the right of the origin at $\epsilon_{t-1} = -\gamma$. Notice that even though ϵ can be large, it cannot drive the conditional variance negative as long as the square is in the specification as well. The Schwert (1990) specification will give this formulation as well.

A closely related model which allows the minimum point of the News Impact Curve to depend upon the standard deviation, is called the Non-linear asymmetric GARCH or NGARCH. It is formulated as:

(11)
$$h_{t} = \omega + \beta h_{t-1} + \alpha (\epsilon_{t-1} + \gamma h_{t-1}^{1/2})^{2}$$

The minimum of the News Impact Curve of the NGARCH model is at $\epsilon_{t-1} = (-\gamma) \cdot h_{t-1}^{1/2}$ which is also to the right of the origin when γ is negative.

Following Nelson's lead, a VGARCH model can also be formulated as:

(12)
$$h_{t} = \omega + \beta h_{t-1} + \alpha (\epsilon_{t-1} / h_{t-1}^{1/2} + \gamma)^{2}$$

where the name VGARCH comes from the representation of $\epsilon/h^{1/2} = v$. Interesting enough, the minimum of the News Impact Curve of the VGARCH model is also at $\epsilon_{t-1} = (-\gamma) \cdot h_{t-1}^{1/2}$. However, the slope of this News Impact Curve is different from that of NGARCH model.

Another important suggestion initially proposed by Glosten, Jaganathan and Runkle (1989) and more recently analyzed by Zakoian (1990), allows both sides of the GARCH News Impact Curve to have different slopes. The model is:

(13)
$$h_{t} = \omega + \beta h_{t-1} + \alpha \epsilon_{t-1}^{2} + \gamma S_{t-1}^{-} \epsilon_{t-1}^{2}, \qquad S_{t}^{-} = 1 \text{ if } \epsilon_{t} < 0,$$
$$S_{t}^{-} = 0 \text{ otherwise}$$

In figure 2, the News Impact Curve is plotted for the GJR model and the non centered AGARCH model.





Finally, there are several papers which have used non-parameteric approaches to the specification and estimation of ARCH models. In some cases, these have started with squared residuals and in others even more generous parameterizations are allowed. See for examples, Pagan and Schwert (1990), Galant, Hsieh and Tauchen (1990), and Gourioux and Monfort (1990). In each case, the non-parameteric procedure forces a short lag structure be used, and in most cases, the heteroskedasticity in the residuals of the squared error regression is not acknowledged in the estimation procedure which leads to inefficient estimation within the class.

III: DIAGNOSTIC TESTS BASED ON THE NEWS IMPACT CURVE

As we have discussed in section II, implicit in any choice of volatility model is a particular News Impact Curve. The standard GARCH model has a News Impact Curve which is symmetric and centered at $\epsilon_{t-1}=0$. That is, positive and negative surprises of the same magnitude would produce the same amount of volatility. Also, larger innovations would create more volatility at a rate proportional to the square of the size of the innovation. If, in fact, a negative innovation casuses more volatility than a positive

innovation of the same size, then the GARCH model will under-predict the amount of volatility following bad news and over-predict the amount of volatility following good news. Furthermore, if large innovations cause more volatility than would be allowed by a quadratic function, then the standard GARCH model will also under-predict volatility after a large shock and over-predict volatility after a small shock. These observations suggest at least three new diagnostic tests for volatility models which we will call the <u>Sign-Bias Test</u>, the <u>Negative-Size-Bias Test</u>, and the <u>Positive-Size-Bias Test</u>. A variety of closely related tests can also be created using the same approach and these tests can be carried out individually or jointly. All these tests can also be applied to other volatility models.

Each of these test statistics examines whether the squared standardized residuals are indeed independent and identically distributed. If there is information in ϵ_{t-1} which could predict these residuals, then the variance process was misspecified. Defining

$$v_t = e_t / \sqrt{\hat{h}_t}$$
, for $y_t = \hat{y}_t + e_t$, $\hat{h}_t =$ estimated conditional variance

and letting z_{t-1} be a vector of measurable functions of the past information set including e_{t-1} in particular, then it is proposed to run the regression:

(14)
$$v_t^2 = a + z_{t-1}b + v_t$$
.

Intuitively, if the model is correctly specified, then b=0 and ν_t is iid. Thus t-statistics and F-statistics on b should have the familiar limiting distributions. By selecting different measures of z, the different tests are constructed. By testing one variable at a time, tests are formulated against a particular alternative, and by allowing several z's, joint tests are constructed. The exact limiting distribution will be discussed after the tests are introduced.

As the names suggest, the Sign-Bias Test focuses on the different impacts that positive and negative innovations have on volatility which are not predicted by the null volatility model. The Negative-Size-Bias Test focuses on the different impacts that large and small <u>negative</u> innovations would have on volatility which are not predicted by the null model. The Positive-Size-Bias Test focuses on the different impacts that large and small <u>positive</u> innovations would have on volatility which are not explained by the null model. It is important to distinguish between positive and negative innovations while examining the "size" effect as an important piece of bad news might have very different impact on volatility than an important piece of good news.

Let S_t^- be a dummy variable that takes a value of 1 if ϵ_{t-1} is negative and zero otherwise. The <u>Sign-Bias Test Statistic</u>, is defined as the t-ratio for the coefficient b in the regression equation:

(15)
$$v_t^2 = a + b \cdot S_t^- + z_{t-1}^- \gamma + \nu_t$$

where there may be one or more other variables z_{t-1} in the regression.

The <u>Negative Size-Bias test statistic</u>, is defined as the t-ratio of the coefficient b in the OLS regression:

(16)
$$\mathbf{v}_t^2 = \mathbf{a} + \mathbf{b}\mathbf{S}_t^- \mathbf{\epsilon}_{t-1} + \mathbf{z}_{t-1}\gamma + \nu_t$$

This test examines whether the more extreme negative ϵ 's are associated with more extreme biases. The corresponding <u>Positive Size-Bias</u> test statistic is defined as the t-ratio of the coefficient b in the same regression equation with $S_t^+ \equiv 1-S_t^-$ as

(17)
$$v_t^2 = a + bS_t^+ \epsilon_{t-1} + z_{t-1}\gamma + \nu_t$$

Alternatively, we can examine even more extreme values of ϵ for particular biases. One approach is to use a variable such as $S_t \tilde{\epsilon}_{t-1}^2$. Another is to define the order statistics of ϵ_{t-1} so that D_t^{α} is the α^{th} percentile of the set of $\{\epsilon_t\}$. For example, regressions including either $S_t \tilde{\epsilon}_{t-1}^2$ or D_t^{10} would be particularly sensitive to the volatility following extreme negative innovations.

These diagnostic test statistics can also be used as summary statistics on the raw data to explore the nature of conditional heteroscedasticity in the data series without first imposing a volatility model. In this case, ϵ_t and v_t would simply be defined as follows:

$$\begin{aligned} \epsilon_{t} &\equiv y_{t} - \mu \\ v_{t} &\equiv \epsilon_{t} / \sigma \end{aligned}$$

where μ and σ are the unconditional mean and standard deviation of y_t respectively. Using these ϵ_t 's and v_t 's, the five summary statistics can be computed based on the regression analyses described above.

Finally, the model can be subjected to all of these tests at once by running the regression:

and testing that all the b's are equal to zero. This can be simply TR^2 or the F statistic for the regression. As there is collinearity among these regressors, the most powerful test against some alternative will not be the joint test. In practice, the use of the first three of these terms seems adequate to reveal the biases in a wide class of heteroskedastic functions for the data set in this study.

In this paper four test statistics will be reported for each model. The regression

(18)
$$v_t^2 = a b_1 S_t^- + b_2 S_t^- \epsilon_{t-1} + b_3 S_t^+ \epsilon_{t-1} + \nu_t$$

is computed and the t-ratios for b_1 , b_2 and b_3 are called the <u>sign bias</u>, the <u>negative size</u> <u>bias</u> and the <u>postitive size bias</u> tests respectively. The <u>joint test</u> is the F statistic from ;this regression which is presented along with its p-value. Finally, three other versions are computed but only reported when they reveal a different behavior. To test the extremes of the size bias, the D¹⁰ and D⁹⁰ are entered into (18); alternatively D⁵ and D⁹⁵, are added to (18); finally rather than the order statistics, $S_t^- \epsilon_{t-1}^2$ and $S_t^+ \epsilon_{t-1}^2$ are added to (18). Each of these regressions has 5 coefficients. If the p value is smaller than the joint one for (18), this fact is noted.

The exact asymptotic distribution of the tests can be derived by considering the LM test as presented in Godfrey(1979) or Engle(1984). Suppose

$$\mathbf{h}_{t} = \mathbf{h}(\mathbf{x}_{t-1}\gamma + \mathbf{z}_{t-1}^{*}\varphi)$$

and we wish to test that $\varphi=0$. Letting $\hat{\gamma}$ be the MLE of γ when $\varphi=0$ and \hat{h}_t be the estimated variance for observation t, and e_t the estimated residual, then the LM test is (19) $\xi_{TM} = TR^2$ of the regression

19) $\xi_{\text{LM}} = \text{TR}^2 \text{ of the regression}$ $e_t^2 / \hat{h}_t = a + x_{t-1} [\hat{h}_t' / \hat{h}_t] c + z_{t-1}^* [\hat{h}_t' / \hat{h}_t] b + \nu_t$

where h_t^{\prime} is the scalar derivative of h_t . When the null hypothesis is true, the scores satisfy a central limit theorem, and the information matrix converges in probability to a constant, then:

$$\xi_{\rm LM} \rightarrow x_{\rm k}^2$$

where k is the number of variables in z_{t-1}^* . The difference between the regressions in (19) and (14) is mainly in the omission of the term in x_{t-1} . As the \mathbb{R}^2 can only be decreased by omitting a set of variables, the distribution of the test statistics from (14) will be less than or equal to the LM test statistic and therefore will have a size less than or equal to the

nominal size. That is, TR^2 from (14) will have a limiting distribution which is less than or equal to a chi square with degrees of freedom equal to the number of variables in z_{t-1} . While it would be easy to construct a test which has the correct asymptotic size, the test would have different variables depending on which model was taken as the null. If the goal is to be certain that the model is able to mimic the observed movements in conditional heteroskedasticity, then the tests described above are natural.

The second implication of the LM derivation is the choice of variables to use for z_{t-1} . To find an optimal test against an alternative z^* , one should use $z = z^* [\hat{h}_t^2 / \hat{h}_t]$. Corresponding to any test z, the implied alternative is $z^* = z/[\hat{h}_t^2 / \hat{h}_t]$.

IV A PARTIALLY NON-PARAMETRIC NEWS IMPACT MODEL

It may well be that no simply parameterized model will pass all of these diagnostic tests. Hence it is appropriate to seek non-parametric models which allow all of these properties to be determined by the data directly. In particular, if the News Impact Curve is the object of the analysis, then it is natural to estimate it non-parametrically. In this section, a simple version of this model is suggested. Because of the long memory characteristic of most variance processes, the decay parameter is specified parametrically. This mixture of parametric and non-parametric parts is labeled partially parametric to distinguish it from Engle and Gonzalez(1990).

Let the range of $\{\epsilon_t\}$ be divided into m intervals with break points τ_i . Let m⁻ be the number of intervals less than zero and m⁺ be the number of positive intervals so that $m = m^+ + m^-$. Denote these boundaries by the numbers $\{\tau_{-m}^-, ..., \tau_{-1}, \tau_0, \tau_1, ..., \tau_m^+\}$. These intervals need not be equal in size and there need not be the same number on each side of τ_0 . For convenience and the ability to test symmetry, $\tau_0 = 0$ is a natural selection. Now define

$$P_{it} = 1 \text{ if } \epsilon_{t-1} > \tau_i$$
, and $N_{it} = 1 \text{ if } \epsilon_{t-1} < \tau_{-i}$.

Then a piecewise linear specification of the heteroskedasticity function is

(20)
$$\mathbf{h}_{t} = \omega + \beta \mathbf{h}_{t-1} + \sum_{i \ge 0}^{m^{+}} \theta_{i} \mathbf{P}_{i}(\epsilon_{t-1} - \tau_{i}) + \sum_{i \ge 0}^{m^{-}} \delta_{i} \mathbf{N}_{i}(\epsilon_{t-1} - \tau_{-i})$$

This functional form is guaranteed to be continuous and is really a linear spline with knots at the τ_i . Between 0 and τ_1 the slope will be θ_0 while between τ_1 and τ_2 it will be $\theta_0 + \theta_1$, and so forth. Above $\tau_m +$, the slope will be the sum of all the θ 's. Clearly, the shape will be monotonic if the partial sums at each point are of the same sign.

As the sample becomes larger, it is possible and desirable to increase m to obtain more resolution. This is an example of the method of sieves approach to non-parametric estimation. A larger value of m can be interpreted as a smaller bandwidth which will give lower bias and higher variance to each point on the curve. If m is increased slowly as a function of sample size, the procedure should asymptotically give a consistent estimate of any News Impact Curve. In this case however, the rate of convergence and the standard errors may be different from standard maximum likelihood results. On the other hand, if m is held fixed, then the estimator will only produce a consistent estimate of the News Impact Curve if (20) is correctly specified. In such a case, the standard errors will be given in their usual form.

It should of course be pointed out that although the specification in (20) is capable of generating a wide range of News Impact Curves, it is very simple with respect to the impact of older information. All information is assumed to decay in an exponential fashion with decay rate β . News affects volatility in the same way in the long run as in the short run. Obviously other terms could be added but this would substantially increase the computaional complexity.

Two simple approaches to choosing the τ_i could be used. They could be unequally spaced based on for example the order statistics, or they could be equally spaced. In the example here, equally spaced bins were used with break points at σ i for $i = 0,\pm 1,\pm 2,\pm 3,\pm 4$ where σ is the unconditional standard deviation of the dependent variable. Thus:

(21)
$$\mathbf{h}_{t} = \omega + \beta \mathbf{h}_{t-1} + \sum_{i \ge 0}^{m} \theta_{i} \mathbf{P}_{it-1}(\epsilon_{t-1} - i\sigma) + \sum_{i \ge 0}^{m} \delta_{i} \mathbf{N}_{it-1}(\epsilon_{t-1} + i\sigma)$$

 $m^+ = m^- = 4$ so that there are 10 coefficients in the News Impact Curve.

Figure 3 below gives an example of the graph of a Partially Non-parametric or PNP News Impact Curve.





V: ESTIMATION OF JAPANESE STOCK RETURN VOLATILITY: 1980-1988

To compare and demonstrate the empirical properties of some of the above mentioned volatility models, we apply these models to the daily returns series of the Japanese TOPIX index. The data were obtained from the PACAP Databases provided by the Pacific Basin Capital Market Research Center at the University of Rhode Island. In this section, we will report our estimation and testing results for the parametric models for the full sample period from January 1, 1980 to December 31, 1988. In the next section, we estimate conditional volatility and the News Impact Curve using a non-parametric approach, and compare the News Impact Curve obtained from the non-parametric method to those obtained from the various parametric volatility models. In section seven, we check the robustness of our results by reestimating some of our models using a shorter sample period from January 1, 1980 to September 30, 1987.

As our focus is on the conditional variance rather than the conditional mean, we concentrate on the unpredictable part of the stock returns obtained through a procedure similar to the one in Pagan and Schwert (1990). The procedure involves a Day-of-the-week effect adjustment and an autoregressive regression removing the predictable part of the return series.

Let y_t be the daily return of the TOPIX index for day t. We first regressed y_t on a constant and five day-of-the-week dummies (for Tue, Wed, Thur, Fri, and Sat) to get the residual, u_t . The u_t was then regressed against a constant and u_{t-1} , ..., u_{t-6} to obtained the residual, ϵ_i , which is our unpredictable stock return data.

The results for the above adjustment regressions and some summary statistics for our unpredictable stock return series are reported below: Day-of-the-week effect adjustment:

$$\begin{aligned} \mathbf{y}_t &= -0.0119 - 0.0907 \cdot \mathrm{TUE}_t + 0.2025 \cdot \mathrm{WED}_t + 0.0953 \cdot \mathrm{THU}_t \\ & (0.039) \quad (0.055) \quad (0.055) \\ & + 0.1100 \cdot \mathrm{FRI}_t + 0.1629 \cdot \mathrm{SAT}_t + \mathbf{u}_t \\ & (0.055) \quad (0.060) \end{aligned}$$

(standard errors in parentheses)

Autocorrelation adjustment:

$$\begin{aligned} \mathbf{u}_{t} &= -0.0002 + 0.1231 \cdot \mathbf{u}_{t-1} - \frac{0.0802 \cdot \mathbf{u}_{t-2}}{(0.02)} + \frac{0.0456 \cdot \mathbf{u}_{t-3}}{(0.02)} \\ &- \frac{0.0526 \cdot \mathbf{u}_{t-4}}{(0.02)} + \frac{0.0772 \cdot \mathbf{u}_{t-5}}{(0.02)} - \frac{0.0706}{(0.02)} \mathbf{u}_{t-6} + \epsilon_{t} \end{aligned}$$

(standard errors in parentheses)

Summary statistics for the unpredictable stock returns:

Mean:	0.0000
Variance:	.6397
Coefficient of Skewness:	-1.8927
Coefficient of Kurtosis:	71.3511
Ljung Box (12) for the levels:	1.3251
Ljung Box (12) for the squares:	406.6700
Sign-bias test:	-6.26
Negative size-bias test:	-20.3
Positive size-bias test:	1.56
Joint test:	139.0
	[0.000]
Number of observations:	2532

From the Ljung Box test statistic for 12-order serial correlation for the levels reported above, there is no significant serial correlation left in the stock returns series after our adjustment procedure. The coefficient of skewness and the coefficient of kurtosis indicate that the unpredictable stock returns, the ϵ_t 's, have a distribution which is skewed to the left and very fat tailed. Furthermore, the Ljung Box test statistic for 12-order serial correlations in the squares strongly suggests the presence of time-varying volatility. The sign-bias, negative size-bias and positive size-bias test statistics introduced in the last section are also computed. The sign bias and negative size bias tests are both highly significant as t-statistics with one degree of freedom. The positive size bias test is not particularly significant, although if the size term were dropped, it would be significant.

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These statistics strongly indicate that the value of ϵ_{t-1} influences today's volatility. Positive innovations appear to increase volatility regardless of the size, while large negative innovations cause more volatility than small ones. The other three regressions which respectively add the 5% and 95% dummy variables, the 10% and 90% dummy variables and the $\pm \epsilon_{t-1}^2$ terms all give lower F statistics (the latter is the highest) and all have p values of 0.0000.

Using the unpredictable stock returns series as the data series, the standard GARCH(1,1) model as well as five other parametric models from the first section which are capable of capturing the leverage effect and the size effect are estimated. They are: the Exponential-GARCH(1,1), the Asymmetric-GARCH(1,1), the VGARCH(1,1), the Nonlinear-Asymmetric-GARCH(1,1) and the Glosten-Jaganathan-Runkle (GJR) model. The estimations are performed using the Bollerslev-Wooldridge Quasi Maximum Likelihood approach. The adequacy of these models is then checked using the sign-bias, the negative size-bias and the positive size-bias tests we have introduced in the last section. The estimation and diagnostic results for each of these models are presented and discussed one by one below. As a convention, the asymptotic standard errors are reported in brackets (\cdot) and the Bollerslev-Wooldridge robust standard errors are reported in square brackets $[\cdot]$.

The GARCH(1,1) model

$h_{\star} = 0.0238$	+ 0.6860 · h.	$ \begin{array}{c} 1 + 0.3299 \cdot \epsilon_{t-1}^2 \\ (0.008) \\ [0.097] \end{array} $
(0.003)) (0.011) ⁽⁻	$(0.008)^{t-1}$
[0.005]	[0.059]	[0.097]

Log-likelihood=	-2356.03
Sign-Bias test:	-0.34
Negative Size-Bias test:	-2.79
Positive Size-Bias test:	-1.54
Joint test:	4.70
	[.0028]

The GARCH(1,1) model assumes a symmetric news impact curve that centers at $\epsilon_{t-1}=0$. It allows big innovations to produce more volatility than small ones following a quadratic function. The significance of the parameters corresponding to the h_{t-1} term and the ϵ_{t-1}^2 confirms the existence of autoregressive conditional heteroscedasticity. However, the model does not distinguish between positive and negative innovations and the negative size-bias test statistic indicates that there is a leverage effect in the data not captured by the specification. Apparently big negative innovations cause more volatility than the GARCH(1,1) model can explain. None of the other three regressions had a lower p value although the one with $\pm \epsilon_{t-1}^2$ had the same p value.

The Exponential-GARCH(1,1) model

Joint test:

$\begin{aligned} \ln h_t &=0668 + 0.9012 \cdot \ln h_{t-1} \\ & (0.010) & (0.007) \\ & [0.020] & [0.022] \\ & + 0.4927 \cdot \left[\frac{ \epsilon_{t-1} }{h_{t-1}^{1/2}} - \frac{ \epsilon_{t-1} }{h_{t-1}^{1/2}} \right] \end{aligned}$	$ \begin{bmatrix} \frac{2}{\pi} \end{bmatrix}^{1/2} - \underbrace{\begin{array}{c} 0.1450}_{\begin{array}{c} 0.011\\ 0.011 \end{array}} \cdot \underbrace{\begin{array}{c} \epsilon_{t-1} \\ h_{t-1}^{1/2} \end{array}}_{t-1} $
Log-likelihood=	2344.03
Sign-Bias test:	10
Negative Size-Bias test:	1.92
Positive Size-Bias test:	62

The Exponential-GARCH model introduced by Nelson (1990) explicitly allows negative and positive innovations to have different impacts on volatility. The significantly negative coefficient corresponding to the term $\epsilon_{t-1}/h_{t-1}^{1/2}$ confirms the presence of the leverage effect. The "exponential" nature of the model also allows big innovations to have much more impact on volatility than small innovations. Although the negative size bias test borders on significance, the tests indicate that the EGARCH model is quite successful in capturing the impacts of news on volatility. In the diagnostic test with $\pm \epsilon_{t-1}^2$, the p value

1.36 [.254] drops to .21 and the negative size bias test becomes significant with a t-ratio of -2.56.

The Asymmetric-GARCH(1,1) model

h _t =	0.0216 + (0.003) [0.005]	$0.6896 \cdot h_{t-1} + (0.012) = [0.055]$	$-0.3174 \cdot (\epsilon_{t-1} - (0.009))$ [0.088]	$(0.1108)^2$ (0.017) [0.030]
	Log—likel Sign—Bia Negative Positive S Joint test	s test: Size–Bias test: Size–Bias test:		345.12 -1.49 -2.77 -1.49 3.38 [.017]

The Asymmetic-GARCH model manages to capture the leverage effect by allowing the news impact curve to center on an non-zero ϵ_{t-1} . The model is attractive as it nests the standard GARCH(1,1) model. Given the same ARCH and GARCH parameters, it offers the same unconditional variance as the standard GARCH(1,1) model but a higher unconditional fourth moment. The new intercept is significantly greater than zero. While the asymmetric GARCH model did correct the sign-bias nature of the standard GARCH model as indicated by the insignificant sign-bias test statistics, the model seems to give too little weight to big negative innovations. The significant negative size-bias test statistic indicates that big negative innovations cause more volatility than the model can explain.

The VGARCH(1,1) model

h. =	0.0192 -	+ 0.6754 · h₄	$-1 + 0.1508 \cdot (\epsilon_{t-1})$	$(h_{+1}^{1/2} - 0.1458)^2$
ι	(0.005)	(0.014)	$(0.004)^{t-1}$	(0.031)
	[0.013]	[0.071]	[0.047]	[0.052]

Log-likelihood= Sign-Bias test: Negative Size-Bias test: Positive Size-Bias test: Joint test: -2424.63 -1.49 -5.20 -0.09 9.33 [.000] The VGARCH model takes virtually the same form as the Asymmetric GARCH model but with the normalized residuals replacing the residuals. The modification reflects the alternative approach which consider the normalized residual rather than the residual itself as "news". Like the Asymmetric GARCH model, the VGARCH model allows a news impact curve that centers on a non-zero ϵ_{t-1} . This again is highly significant. The significant negative size-bias test statistic indicates that the VGARCH model suffers from the same problem as the Asymmetric GARCH model. It fails to capture entirely the fact that big negative innovations do cause much more volatility than small negative innovations.

The Nonlinear-Asymmetric GARCH(1,1) model

	$ \begin{smallmatrix} 0.2515\cdot(\epsilon_{t-1} & -0.2683\cdot\sqrt{\mathbf{h}_{t-1}})^2 \\ (0.008) & (0.036) \\ [0.083] & [0.061] \end{smallmatrix} $
Log-likelihood= Sign Bias test: Negative Size-Bias test: Positive Size-Bias test: Joint test:	-2335.34 -1.49 -3.12 99 3.63 [.0125]

The Non-linear Asymmetric GARCH(1,1) model gives more weight to extreme innovations by allowing the minimum of the News Impact Curve to depends on the past conditional standard deviation. While the model passes the Sign-bias test and the Positive size-bias test, the Negative size-bias test statistic is significant at the 1% level indicating that big negative innovations still product more volatility than the model can explain.

The Glosten-Jaganathan-Runkle (GJR) model

	$\begin{smallmatrix} 0.1672 \cdot \epsilon_{t-1}^2 + 0.2636 \cdot S_{t-1}^- \epsilon_{t-1}^2 \\ (0.018) & (0.020) \\ [0.036] & [0.102] \end{smallmatrix}$
Log-likelihood= Sign-Bias test: Negative Size-Bias test: Positive Size-Bias test: Joint test:	-2333.11 -1.08 99 99 1.59 [.189]

The GJR model, designed explicitly for the leverage effect, has a News Impact Curve that centers at $\epsilon_{t-1}=0$ but has a much steeper slope for negative ϵ_{t-1} 's. The significance of the coefficient corresponding to the term $S_{t-1}^- \epsilon_{t-1}^2$ confirms the existence of the leverage effect. There is no evidence of unexplained sign or size bias in the positive or negative side.

Overall, the Exponential GARCH model and the GJR model seems to outperform all other models in capturing the dynamic behavior of the Japanese stock return volatility with the GJR model having a higher loglikelihood. To further our understanding about these different volatility models, some summary statistics including the mean, standard deviation, minimum, maximum, skewness and kurtosis are produced for each of the estimated conditional variance series. They are report as follows:

	Summary Statistics					
	Mean	Std.Dev.	Min.	Max.	Skew.	Kurto.
$\epsilon_{\rm t}^2$	0.6397	5.366	2.8e-8	236.60	37.038	1543.47
h_t^{GARCH}	0.7483	3.124	0.0842	90.83	21.173	523.20
h ^E GARCH t	0.8669	10.555	0.0491	485.27	40.843	1799.19
h ^{AGARCH}	0.7367	3.047	0.0807	87.78	21.014	515.64
h t VGARCH	0.5243	0.674	0.0943	20.16	15.279	373.39
h t NGARCH	0.6961	2.574	0.0847	64.47	18.215	392.61
${}^{h}_{t}^{GJR}$	0.7561	3.492	0.0885	104.21	21.950	559.01

The conditional variance series produced by our winners: the EGARCH model and the GJR model have the highest variation over time. The estimated conditional variance ranges from a low of 0.0491 to a high of 485.27 compared to 0.0842 and 90.83 under the standard GARCH model. The standard deviation of the EGARCH conditional variance, 10.555 is more than three times that of the standard GARCH model. It also has a much more skewed and fat tailed distribution than the other conditional variance series or even the squared innovations.

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VI: PARTIALLY NON-PARAMETRIC ARCH ESTIMATION

The News Impact Curve is also estimated by fitting a partially non-parametric model of the form given in (19). The exact specification and the estimation results are reported below:

Partially Non-parametric ARCH (PNP)

	$(\log L = -2310.72)$
$^{+\ 0.0897 \cdot P}_{\substack{(0.014)\\[0.043]}} t-1} \epsilon_{t-1}$	$^{+\ 0.2269 \cdot P_{1t-1}(\epsilon_{t-1} - \sigma)}_{\substack{(0.088)\\[0.172]}}$
$^{+0.6666 \cdot P}_{egin{array}{c} 0.353 \\ 0.720 \end{bmatrix}} t_{t-1}^{(\epsilon_{t-1}-2\sigma)}$	$\substack{-3.7664 \cdot \mathbf{P}_{3t-1}(\epsilon_{t-1} - 3\sigma) \\ (1.096) \\ (1.991]}$
$^{+\ 3.6915\cdot P}_{\substack{4t-1}(\epsilon_{t-1}-4\sigma)}_{\substack{(1.540)\\[2.327]}}$	
${}^{0.1536 \cdot N}_{egin{smallmatrix} 0.014 \ 0.053 \end{bmatrix}} {}^{\epsilon}_{t-1}$	$egin{array}{c} -0.3312 \cdot \mathbf{N}_{1t-1}(\epsilon_{t-1}+\sigma) \\ (0.093) \\ (0.203] \end{array}$
$\begin{array}{c} -3.1194 \cdot \mathrm{N}_{2t-1}(\epsilon_{t-1}+2\sigma) \\ (0.278) \\ [4.143] \end{array}$	$^{+7.3481\cdot}_{\substack{(0.959)\\[8.699]}}^{*}_{3t-1}(\epsilon_{t-1}+4\sigma)$
(1.679) (1.679) $(5.769]$	

where, σ is the unconditional standard deviation of ϵ_t

The specification is a piecewise linear model with kinks at ϵ_{t-1} equal to 0, σ , 2σ , 3σ and 4σ . If we compare the values of the coefficients corresponding to the terms $P_{it-1}(\epsilon_{t-1}-i\sigma)$ i=0,1,2 to their counterparts: $N_{it-1}(\epsilon_{t-1}+i\sigma)$ i=0,1,2, we can see that negative ϵ_{t-1} 's do cause more volatility than positive ϵ_{t-1} 's of equal absolute size. Moreover, the rate of increase in volatility as we move towards ϵ_{t-1} 's with bigger absolute size is higher for negative ϵ than for the positives. Hence, there seems to be a sign or leverage effect as well as a size effect that differs for negative and positive ϵ 's. The estimated parameter values for the terms $P_{it-1}(\epsilon_{t-1}-i\sigma)$ and $N_{it-1}(\epsilon_{t-1}+i\sigma)$ for i=3,4 have somewhat unexpected signs and magnitudes. Since these terms are for the extreme ϵ 's, they might be driven by only a few outliers. Indeed, even though they are significant under the traditional asymptotic standard errors, they are by all means insignificant under the Bollerslev-Wooldridge robust standard errors. The non-parametric estimation results thus seem to indicate that the true News Impact Curve probably has a steeper slope in the negative side than in the positive side.

A comparison of the news impact curve implied by the various volatility models and that corresponding to this piecewise linear non-parametric model is performed by computing, for each of these models, the implied volatility level at several prespecified values for ϵ_{t-1} under the assumption that $h_{t-1} = \sigma = 0.63966$. The results are summarized in the table below:

€t−1	h_t^{GAR}	CH h ^{EGAI}	RCH _h AGA	RCH _h VGA	RCH _h NGA	RCH _h GJR	h_t^{PNP}
—10.	33.45	1225.1	32.91	24.58	26.73	43.55	12.793
—5.	8.71	22.739	8.753	6.623	7.323	11.245	4.061
$\begin{array}{r} -2.5 \\ -2.0 \\ -1.0 \\ -0.5 \\ 0.0 \\ 0.5 \\ 1.0 \\ 2.0 \\ 2.5 \end{array}$	$\begin{array}{c} 2.524 \\ 1.782 \\ 0.793 \\ 0.545 \\ 0.463 \\ 0.545 \\ 0.793 \\ 1.782 \\ 2.524 \end{array}$	$\begin{array}{c} 3.098\\ 2.079\\ 0.937\\ 0.629\\ 0.422\\ 0.525\\ 0.652\\ 1.007\\ 1.251\end{array}$	$\begin{array}{c} 2.626\\ 1.877\\ 0.854\\ 0.581\\ 0.467\\ 0.511\\ 0.714\\ 1.596\\ 2.275\end{array}$	$\begin{array}{c} 2.065\\ 1.507\\ 0.745\\ 0.541\\ 0.454\\ 0.486\\ 0.635\\ 1.287\\ 1.790\end{array}$	$\begin{array}{c} 2.337\\ 1.717\\ 0.855\\ 0.612\\ 0.495\\ 0.504\\ 0.639\\ 1.286\\ 1.797\end{array}$	$\begin{array}{c} 3.167\\ 2.198\\ 0.906\\ 0.583\\ 0.475\\ 0.517\\ 0.642\\ 1.144\\ 1.520\end{array}$	$\begin{array}{r} 3.533\\ 2.470\\ 0.736\\ 0.593\\ 0.517\\ 0.561\\ 0.652\\ 1.235\\ 1.348\end{array}$
5.0	8.710	3.710	8.050	6.073	6.243	4.655	1.038
10.0	33.453	32.616	31.503	23.480	24.566	17.195	5.579

The News Impact Curves

If we first confine ourselves to ϵ_{t-1} in the range (-2.5,2.5), we can see that the standard GARCH model tends to understate h_t for large negative ϵ_{t-1} 's and overstate h_t for large positive ϵ_{t-1} 's relative to the EGARCH as was indicated by our previous test statistics. These are also true for the AGARCH, VGARCH and NGARCH models. Among all six parametric models, the EGARCH and the GJR have News Impact Curves that are closest to the one suggested by the non-parametric estimation. Now if we consider the very extreme values for ϵ_{t-1} , then we can see that the EGARCH and the GJR are indeed very different. In fact, because of the exponential functional form, the EGARCH produces a ridiculously high h_t of 1225.1 for an ϵ_{t-1} equal to -10 which is about three thousand times the value of the unconditional variance. Since stock market volatility wasn't that high in Japan after the 1987 crash, we feel that the EGARCH might be too extreme in the tails. The GJR model which also has a higher log-likelihood than the EGARCH might be a more reasonable model to use.

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VII: SUBSAMPLE ROBUSTNESS CHECK

To judge the sensitivity of our results to the extreme observations around the 1987 crash, we have repeated part of our our analysis for the subsample period from January 1, 1980 to September 30, 1987 excluding the crash. The results for the day-of-the-week and autocorrelation adjustments as well as some summary statistics for the residuals are reported below:

Day-of-the-week effect adjustment:

$$y_{t} = \begin{array}{c} 0.0162 - 0.1088 \cdot \text{TUE}_{t} + 0.1412 \cdot \text{WED}_{t} + 0.0682 \cdot \text{THU}_{t} \\ (0.035) & (0.049) & (0.049) & (0.049) \\ + 0.1008 \cdot \text{FRI}_{t} + 0.1411 \cdot \text{SAT}_{t} + u_{t} \\ (0.049) & (0.053) \end{array}$$

(standard errors in parentheses)

Autocorrelation adjustment:

(standard errors in parentheses)

Summary Statistics for ϵ :

Mean: Variance: Coefficient of Skewness: Coefficient of Kurtosis: Ljung-Box(12) for the levels: Ljung-Box(12) for the squares:	0.0000 0.4302 0.0947 8.7135 5.9197 540.8552
Sign-bias test: Negative size-bias test: Positive size-bias test: Joint test:	-2.3 -13.9 5.64 77.0 [.000]
Number of observations:	2192

The Ljung-Box(12) statistic for the squares strongly suggest the existence of autocorrelation in the squared residuals and hence time-varying conditional volatility of the autoregressive type. The Sign-bias test statistic is significant and the two size-bias test statistics are also highly significant with the Negative size-bias test statistics having a higher value. It is therefore highly probable that there is a size effect (big news cause more volatility than small news) which is stronger for bad news than for good news. Given the superiority of the EGARCH and the GJR over the other asymmetric volatility models, we have therefore repeated our estimation for the standard GARCH, the EGARCH and the GJR only. The results are reported below:

The GARCH(1,1) model

Log-likelihood=	-1829.50
Sign-bias test:	.05
Negative size-bias test:	1.83
Positive size-bias test:	84
Joint test:	2.51
	[.057]

The Exponential-GARCH(1,1) model

$$\begin{split} \ln \mathbf{h}_{t} &= -.0350 + 0.9579 \cdot \ln \mathbf{h}_{t-1} \\ \begin{pmatrix} 0.007 \\ 0.014 \\ 0.014 \\ 0.019 \\ 0.037 \\ \end{bmatrix} \left[\frac{|\epsilon_{t-1}|}{\mathbf{h}_{t-1}^{1/2}} - \left(\frac{2}{\pi}\right)^{1/2} \right] - \underbrace{0.0615}_{\begin{pmatrix} 0.010 \\ 0.010 \\ 0.024 \\ \end{bmatrix}} \frac{\epsilon_{t-1}}{\mathbf{h}_{t-1}^{1/2}} \end{split}$$

Log-likelihood=	-1822.30
Sign-bias test:	47
Negative size-bias test:	-2.14
Positive size-bias test:	03
Joint test:	1.75
	[.154]

The Glosten-Jaganathan-Runkle (GJR) model

	$ \begin{array}{c} 0.1130 \cdot \epsilon_{t-1}^2 + 0.1048 \cdot S_{t-1}^- \epsilon_{t-1}^2 \\ (0.014) & (0.019) \\ (0.023) & [0.038] \end{array} $
Log-likelihood= Sign-bias test: Negative size-bias test: Positive size-bias test: Joint test:	-1819.23 18 -1.24 29 .75 [.519]

Several points are worth special notice in the above results. First, the parameter corresponding to the $\epsilon_{t-1}/h_{t-1}^{1/2}$ term in the EGARCH and the parameter corresponding to the $S_{t-1}^- \epsilon_{t-1}^2$ term in the GJR are both highly significant even under the Bollerslev-Wooldridge t-test. Second, the Joint test for the standard GARCH model is nearly significant while those for the EGARCH and GJR are not. The log-likelihood of the EGARCH and the GJR are substantially higher than that of the standard GARCH. All of these results point to the presence of a leverage effect in the data. In terms of the size effect, the positive size-bias test is insignificant for all three models indicating that there is not much size effect for positive innovations. However, the negative size-bias test statistics is marginally significant for the standard GARCH and significant for the EGARCH but insignificant for the GJR. The failure of the EGARCH to capture the size effect is probably due to the fact that the quadratic dominates the exponential for small ϵ 's and that the Japanese stock market was quite calm before the 1987 crash. The only model that seems to do well in both normal and abnormal times is the GJR model which also has the higher log-likelihood in both periods.

VIII: SUMMARY AND CONCLUSION

This paper has introduced the News Impact Curve as a standard measure of how news is incorporated into volatility estimates. In order to better estimate and match News Impact Curves to the data, several new candidates for modelling time varying heteroskedasticity are introduced and contrasted.. These models allow several types of asymmetry in the impact of news on volatility. Furthermore, some new diagnostic tests are presented which are designed to determine whether the volatility estimates are adequately representing the data. Finally, a partially non-parametric model is suggested which allows the data to determine the News Impact Curve directly.

These new models are fitted to daily Japanese stock returns from 1980-1988. All the models find that negative shocks introduce more volatility than positive shocks and that this is particularly apparent for the largest shocks. The diagnostic tests however indicate that in many cases, the modelled asymmetry is not adequate. The best models are ones proposed by Glosten Jaganathan and Runkle(GJR) and Nelson's(1990) EGARCH.

The partially non-parametric (PNP) ARCH model is then fitted to the data and reveals much the same behavior. For reasonable values of the surprises, the volatility forecast by EGARCH, GJR and PNP, are rather similar. For more extreme shocks, they differ dramatically. It turns out that the standard deviation, skewness and kurtosis of EGARCH and GJR are all greater than the other models, and in some cases greater than even the squared returns.

When the same analysis is carried out excluding the October 1987 crash, the results are less dramatic but roughly the same. The evidence agains the symmetric GARCH model is not as strong, but the asymmetric models GJR and EGARCH again dominate. In this case, there is also evidence agains the EGARCH and the GJR model appears the best.

REFERENCES

Bera, A. and S. Lee (1989), "On the Formulation of a General Structure for Conditional Heteroskedasticity," unpublished manuscript, Department of Economics, University of Illinois at Urbana-Champaign.

Black, Fisher (1976), "Studies in Stock Price Volatility Changes," PROCEEDINGS OF THE 1976 BUSINESS MEETING OF THE BUSINESS AND ECONOMICS STATISTICS SECTION, AMERICAN STATISTICAL ASSOCIATION, 177–181.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," JOURNAL OF ECONOMÉTRICS, 31, 307-327.

Bollerslev, T. and J. Wooldridge (1989), "Quasi Maximum Likelihood Estimation of Dynamic Models with Time Varying Čovariances," unpublished manuscript, Department of Economics, MIT.

Bollerslev, T., R. Chou, N. Jayaraman, and K. Kroner (1990), "ARCH Modeling in Finance: A Selective Review of the Theory and Empirical Evidence, with Syggestions for Future Research," Memo, Northwestern University.

Engle, R. (1982), "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," ECONOMETRICA, 50, 987-1008.

Engle, R. (1990), "Discussion: Stock Volatility and the Crash of 87," REVIEW OF FINANCIAL STUDIES, Volume 3, Number 1, 103-106.

Engle, R. and T. Bollerslev (1986), "Modelling the Persistence of Conditional Variances," ECONOMETRIC REVIEW, 5, 1-50, 81-87.

Engle, R. and C. Mustafa (1989), "Implied ARCH Models from Options Prices," unpublished manuscript, Department of Economics, UCSD. Friedman, B. and K. Kuttner (1990), "Time Varying Risk Perceptions and the

Pricing of Risky Assets," Department of Economics, Harvard University.

Gallant, A.R., D. Hsieh and G. Tauchen (1990), "On Fitting a Recalcitrant Series: The Pound/Dollar Exchange Rate 1974-83," unpublished manuscript, Department of Economics, Duke University.

Geweke, J. (1986), "Modeling the Persistence of Conditional Variances: A Comment," ECONOMETRIC REVIEW, 5, 57-61.

Glosten, L., R. Jagannathan and D. Runkle (1989), "Relationship Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks," unpublished manuscript, J.L. Kellogg Graduate School, Northwestern University.

Gourioux, C. and A. Monfort (1990), "Qualitative Threshold ARCH Models," unpublished manuscript, INSEE.

Higgins, M. and A. Bera (1990), "A Class of Nonlinear ARCH Models," Department of Economics, University of Wisconsin at Milwaukee.

Mihoj, A. (1987), "A Multiplicative Parameterization of ARCH Models," unpublished manuscript, Department of Statistics, University of Copenhagen.

Nelson, D. (1990), "Conditional Heteroskedasticity in Asset Returns: A New Approach," ECONOMETRICA, forthcoming.

Pantula, S.G. (1986), "Modeling the Persistence of Conditional Variances: A Comment," ECONOMETRIC REVIEW, 5, 71–74.

Pagan, A. and G.W. Schwert (1990), "Alternative Models for Conditional Stock Volatility," JOURNAL OF ECONOMETRICS, forthcoming.

Schwert, G.W. (1990), "Stock Volatility and the Crash of 87," REVIEW OF FINANCIAL STUDIES, Volume 3, Number 1, 77-102.

Zakoian, J. (1990), "Threshold Heteroskedasticity Model," unpublished manuscript, INSEE.